Semiconductor Optical Communication
Components and Devices

Lecture 23: Direct Laser Modulation

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http://www.iitk.ac.in/ee/faculty/det_resume/utpal.html
Pulse Modulation of Diode Lasers

At Steady State

\[
dN = \frac{J_0}{qd} u(t) - \frac{N}{\tau_c}
\]

Where \( u(t) \) is a unit step function

A general solution is

\[
N(t) = A e^{-t/\tau_c} + B e^{t/\tau_c} + C
\]

Boundary conditions:

\( N = N_0 \) at \( t = \infty \), \( B = 0 \) & \( C = N_0 \)

and \( N \approx 0 \) at \( t = 0 \) or \( A = -N_0 \)

\[
N(t) = N_0 u(t) \left[ 1 - e^{-t/\tau_c} \right]
\]

For \( J_0 > J_{th} \)

\[
N_{on} \approx N_0 \approx \left( \frac{\tau_c}{qd} \right) J_{th}
\]

\[
\approx \left( \frac{\tau_c J_{th}}{qd} \right) \left[ 1 - \exp\left( -\frac{t_d}{\tau_c} \right) \right] u(t)
\]

\[
t_d = \tau_c \ln \left( \frac{J - qN_i/\tau_c}{J - qN_f/\tau_c} \right)
\]

\( \tau_c \Rightarrow \square \text{ ns}, \quad \tau_p \Rightarrow \square \text{ ps} \)

\[
t_d = \tau_c \ln \left( \frac{J}{J - J_{th}} \right)
\]

\( \tau_p \equiv \text{Photon lifetime} \)
Intensity Modulation of Laser - I

For the frequency response we start with the rate equations:

\[
\frac{dN}{dt} = \frac{J}{qd} - g_1 (N-N_T) \rho_v \delta v - \frac{N}{\tau_c}
\]

\[
d (\rho_v \delta v) = g_1 (N-N_T) \rho_v d\nu - \frac{\rho_v \delta v}{\tau_{ph}} + \gamma \frac{N}{\tau_c}
\]

Where \( \gamma << 1 \)

For small signal analysis consider

\[J = J_0 + J_1 e^{j\omega t}\]

Assume small signal operation.

\[J_1 << J_0\]

At steady state

\[0 = g_1 (N_o - N_T) \rho_{v_0} d\nu_0 - \frac{N_o}{\tau_c} - \frac{\rho_{v_0} d\nu_0}{\tau_{ph}} + \gamma \frac{N_o}{\tau_c}\]
Intensity Modulation of Laser - II

Assume the carrier density and the photon density changes with the current modulation in the same fashion for small signal operation.

Then carrier density is given as \( N = N_0 + N_1 e^{j\omega t} \) Where \( N_0 \bigg| J_0 \gg J_{th} = N_{th} \)

photon density is given as \( \rho_v dv = \rho_{v_0} dv_0 + \rho_{v_1} dv_1 e^{j\omega t} \)

Putting these equations in the differential rate equations and equating to zero the steady state conditions

\[
j \omega N_1 e^{j\omega t} = \frac{J_0 + J_1 e^{j\omega t}}{qd} - g_1 \left( N_0 + N_1 e^{j\omega t} - N_T \right) \left( \rho_{v_0} dv_0 + \rho_{v_1} dv_1 e^{j\omega t} \right) - \frac{N_0 + N_1 e^{j\omega t}}{\tau_c}
\]

and

\[
g_1 \left( N_0 + N_1 e^{j\omega t} - N_T \right) \left( \rho_{v_0} dv_0 + \rho_{v_1} dv_1 e^{j\omega t} \right) = \frac{J_0}{qd} + \frac{J_1 e^{j\omega t}}{qd} - \frac{N_0}{\tau_c} + \frac{N_1 e^{j\omega t}}{\tau_c}
\]

Therefore the ac terms can we written as

\[
j \omega N_1 e^{j\omega t} = \frac{J_1 e^{j\omega t}}{qd} - g_1 N_1 e^{j\omega t} \rho_{v_0} dv_0 - g_1 \left( N_0 - N_T \right) \rho_{v_1} dv_1 e^{j\omega t} + \frac{N_1 e^{j\omega t}}{\tau_c}
\]
Intensity Modulation of Laser - III

\[
\therefore j\omega N_1 + g_1 N_1 \rho_{v_1} dv_0 + g_1 \left( N_o - N_T \right) \rho_{v_1} dv_1 - \frac{N_1}{\tau_c} - \frac{J_1}{qd} = 0
\]

And for the carrier equation

\[
j\omega \rho_{v_1} dv_1 = g_1 N_1 \rho_{v_0} dv_0 + g_1 \left( N_o - N_T \right) \rho_{v_1} dv_1 - \frac{\rho_{v_1} dv_1}{\tau_{ph}} + \gamma \frac{N_1}{\tau_c} = 0
\]

Again \( J = J_0 m_J \) where \( m_J \) is the modulation depth

And solving for \( N_1 \) and \( \rho_{v_1} dv_1 \)

\[
\rho_{v_1} dv = \frac{J_1}{qd} \left( g_1 \rho_{v_0} dv_0 + \frac{\gamma}{\tau_c} \right) / K
\]

where

\[
K = \left[ j\omega + \frac{1}{\tau_c} + g_1 \rho_{v_0} dv_0 \right] \left[ j\omega + \frac{1}{\tau_{ph}} - g_1 \left( N_o - N_T \right) \right] + g_1 \left( N_o - N_T \right) \left( g_1 \rho_{v_0} dv_0 + \frac{\gamma}{\tau_c} \right)
\]
Hence, Frequency Response is given by

\[
\eta_D(\omega) = \frac{\rho v_1 dv_1}{J_1/qd} = \frac{\tau_{ph} \omega_0^2}{J_0 + \tau_c + \tau_{ph} \omega_0^2} \left( \omega_o^2 - \omega^2 \right) + j\omega \left( \frac{1}{\tau_c} \right)
\]

where

\[
\omega_o^2 = \frac{1}{\tau_{ph} \tau_c} \left( 1 + g_1 N_T \tau_{ph} \right) \left( \frac{J_0}{J_{th}} - 1 \right)
\]

\[
\frac{\rho v_1 dv}{J_1/qd}
\]

\[
\frac{\tau_c}{\tau_{ph}} = 1000
\]

\[
J_o = 2J_{th}
\]

\[
J_o = 5J_{th}
\]
Measured Response - I

Typical ringing observed in a Laser pulsed from below threshold

1011010101 bits is the data for which modulation is done
It can be seen that to operate the laser at high frequency it needs to be biased at a high level such that the effect of chirping is less. On the flip side it needs better cooling system and the cost rises.
1. A diode laser is operated at a current $I_o=2I_{th}$. If the Carrier and Photon Lifetimes are 1ns and 10ps, respectively, then estimate the modulation bandwidth of the laser. Given that the transparency carrier concentration is $1.0 \times 10^{18}$ cm$^{-3}$ and $g_1=5\times10^{-7}$ cm$^3$ s$^{-1}$.

2. A 800nm thick DH laser is biased at $0.5I_{th}$ and a pulse current step of $1.0I_{th}$ is applied to the laser at $t=0$. If the Carrier and Photon Lifetimes are 1ns and 10ps, respectively, find the delay time after which the output power reaches the steady state value. Calculate the time after which the output power reaches steady state.

3. Where would you bias a semiconductor laser for high speed direct modulation? What is the penalty paid for this state of operation?