Cascode Amplifier

M1 & M2 are so biased with current source I, such that they remain in saturation.
M2 is further applied a DC voltage Vref. However, gate of M2 is at 0 volt for AC operation.
M1 is the Driver Transistor
CL is capacitive load.
The output resistance of current source is very high → ∞
We wish to find $G_{meff}$ and $G_{oefl}$ of this this cascode stage.

Let $g_m$ and $g_o$ ($\frac{1}{f_o}$) are the transconductance and output conductance of Transistor M1.

Similarly $g_m$ and $g_o$ are for Transistor M2.

By network theorems, small signal current $i$ (dI) is given by

\[ i = G_{meff} \cdot v_{in} + G_{oefl} \cdot v_o \]  

Let $v_{o1}$ be drain voltage of M1. However, this voltage is also `Source` voltage of M2.
For AC signal, \( V_{OS2} = 0 - V_{01} = -V_{01} \)

From current equation (1)

\[
g_{mse} = \frac{i}{v_{in}} \bigg|_{v_{0} = 0} \quad \text{and} \quad g_{mse} = \frac{i}{v_{0}} \bigg|_{v_{in} = 0}
\]

Assume we apply a fixed voltage \( V_{fix} \) at the drain of \( M_2 \) (\( V_0 \) terminal). Then for ac case, \( v_0 = 0 \)

\[ (V_0 - V_s)_{M_2} = d(V_{DS2}) = -V_{01} \]

Then

\[ i = g_{m1} v_{in} + g_{01} (V_{01}) \quad - (2) \]

\[ i = g_{m2} (-V_{01}) + g_{02} (-V_{01}) \quad - (3) \]
From eq (3) \( v_{o1} = \frac{-i}{g_{m2} + g_{o2}} \)

Substituting this in eq (2) as

\[ i = g_{m1} v_{in} + \frac{-g_{o1} i}{g_{m2} + g_{o2}} \]

\[ \therefore i \left( 1 + \frac{g_{o1}}{g_{m2} + g_{o2}} \right) = g_{m1} v_{in} \]

\[ \therefore g_{\text{meff}} = \frac{i}{v_{in}} = \frac{g_{m1}}{(1 + \frac{g_{o1}}{g_{m2} + g_{o2}})} \]
or \[ g_{\text{m eff}} = \frac{g_{m1} (g_{m2} + g_{o2})}{g_{m2} + g_{o1} + g_{o2}} \]

If \( r_o \) of \( M1, M2 \) are large, then \( g_{o1} \) and \( g_{o2} \) are very small.

\[ g_{\text{m eff}} \approx \frac{g_{m1} g_{m2}}{g_{m2}} = g_{m1} \]

Hence we observe that \( g_{\text{m eff}} \) of the cascade stage is almost same as \( g_{m1} \).

Hence if we have normal amplifier without \( M2 \), then transconductance \( g_{m} = g_{m1} \).
Next we evaluate $g_{oef}$. To get $g_{oef}$, from eqn. (1) we have

$$g_{oef} = \frac{i}{v_0} \bigg|_{v_{in} = 0}$$

Circuit looks like as shown in Fig. We do ac analysis and get

$$i = -g_{m2}v_{o1} + g_{o2} (v_{o2} - v_{o1}) \quad (4) \quad v_o = v_{o2}$$

$$i = g_{m1} \cdot 0 + g_{o1} v_{o1} \quad (5)$$

$$\therefore v_{o1} = \frac{i}{g_{o1}}$$

Substituting this in eq. (4)
\[
\therefore i = -\frac{g_m}{g_{o1}} i + g_{o2} v_0 + g_o \left(-\frac{i}{g_{o1}}\right)
\]
\[\alpha i \left(1 + \frac{g_{o2}}{g_{o1}}\right) + (g_m) i = g_{o2} v_0\]
\[
\therefore g_{o\text{eff}} = \frac{i}{v_0} = \frac{g_{o2}}{1 + \frac{g_{o2}}{g_{o1}} + \frac{g_m}{g_{o1}}}
\]
\[
= \frac{g_{o1} g_{o2}}{g_{o1} + g_{o2} + g_m}
\]
\[
\therefore g_{o\text{eff}} = \frac{g_{o1} + g_{o2} + g_m}{g_{o1} g_{o2}}
\]
\[
= \frac{1}{g_{o2}} + \frac{1}{g_{o1}} + \frac{g_m}{g_{o1} g_{o2}}
\]
\[ \gamma_{\text{eff}} = \gamma_0 + \gamma_1 + \gamma_1 \left( \frac{g_m}{g_0} \right) \]

\[ = \gamma_0 + \gamma_2 + A_{\nu_2} \gamma_1 \]

\[ \gamma_{\text{eff}} = \gamma_2 + (1 + A_{\nu_2}) \gamma_1 \]

Voltage Gain \( A_{\nu_2} \) of the Cascade Stage Amplifier is

\[ A_{\nu_{\text{eff}}} = -\frac{g_m}{g_{\text{eff}}} \]

\[ = -g_m \left[ \gamma_2 + (1 + A_{\nu_2}) \gamma_1 \right] \]

\[ = + A_{\nu_2} + A_{\nu_1} + A_{\nu_1} A_{\nu_2} \]

\[ = A_{\nu_1} A_{\nu_2} \quad \text{if} \quad A_{\nu_1} = A_{\nu_2} \]

\[ \text{then} \quad A_{\nu_{\text{eff}}} = A_{\nu_1} \]
Gain Bandwidth Product GBW of Cascode

\[
\frac{G_m \text{eff}}{C_m} = \frac{G_m}{C_m} \rightarrow \text{NO CHANGE}
\]

If \(M_1, M_2\) are identical, then \(g_m = g_{m1} = g_{m2}\)
\(r_o = r_{o1} = r_{o2}\)

Then \(\text{Gain}_{\text{cascode}} = (\text{Gain}_{\text{single stage}})^2\)

For Cascode, Gain Enhances (Boosted) but Bandwidth remains same.

Thus Technology Constraint of Transistor is as if BROKEN.