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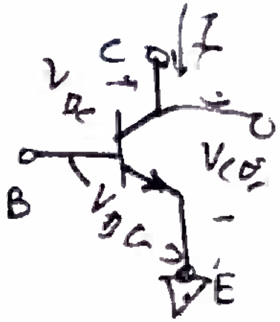
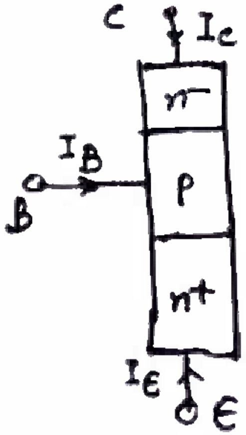
Bipolar Transistor Models





$$I_E = I_B + I_C \quad \text{(Universal)} \qquad —①$$

$$I_C = \alpha I_E \qquad \alpha = \alpha_T \gamma \qquad —②$$

$$\frac{I_C}{I_B} = \beta \qquad \therefore \quad \beta = \frac{\alpha}{1-\alpha} \qquad —③$$

β is called Forward Current Gain

$$I_C = \alpha I_E + I_{CO} \qquad —④$$

$$I_{CO} = I_{CS}(1 - \alpha_F \alpha_R) \qquad —⑤$$

$$V_{CE} = V_{BE} + V_{BC} \qquad —⑥$$

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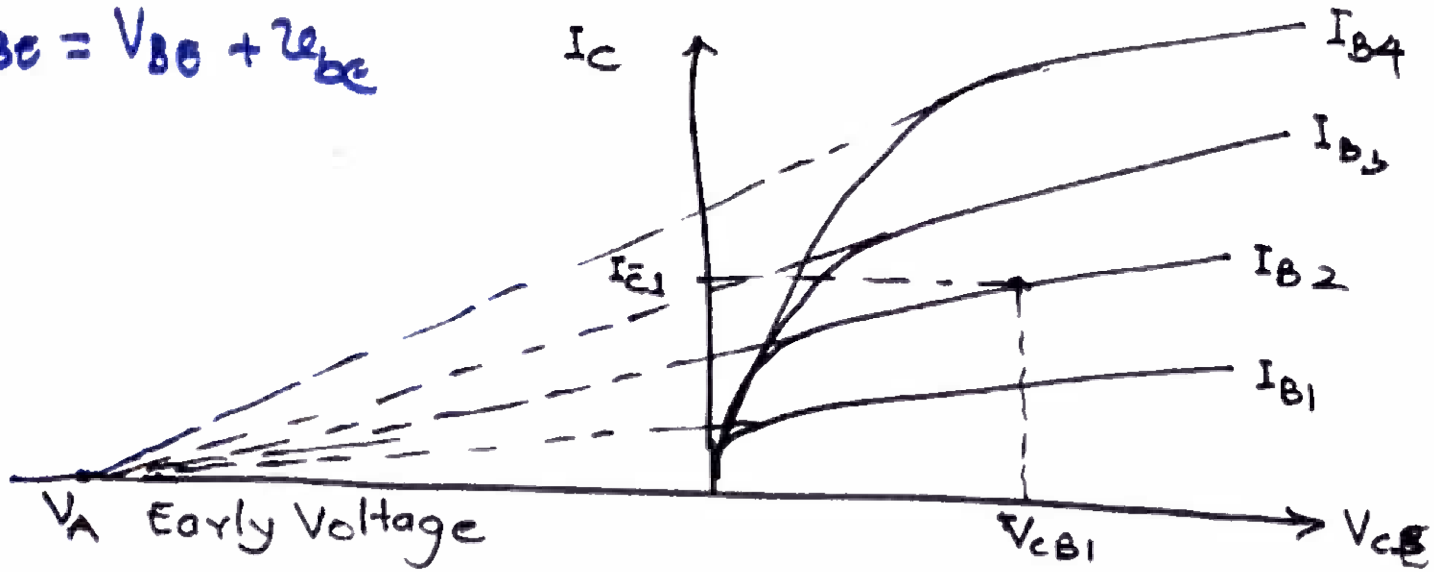
Small-Signal Model of BJT

$$I_b = i_b + I_B$$

Total = Small Signal + DC

$$I_C = i_c + I_C$$

$$v_{BE} = V_{BE} + v_{be}$$



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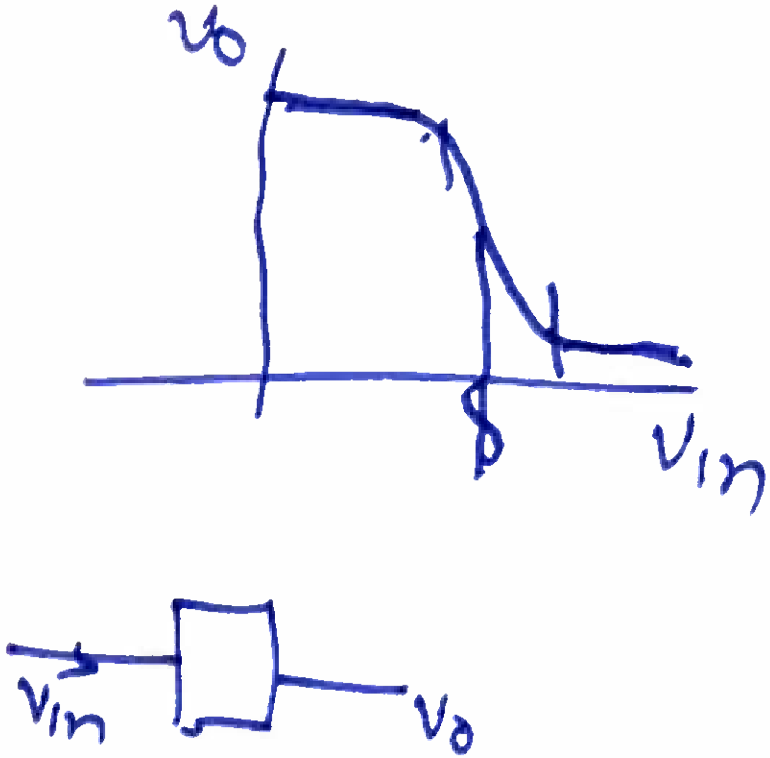

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Small Signal Model (Cont.)

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Most Important parameter of Transistor is

Transconductance $g_m = \dfrac{\partial I_c}{\partial V_{BE}}$ — (1)

$(I_c = I_s \exp(V_{BE}/V_T))$

We have

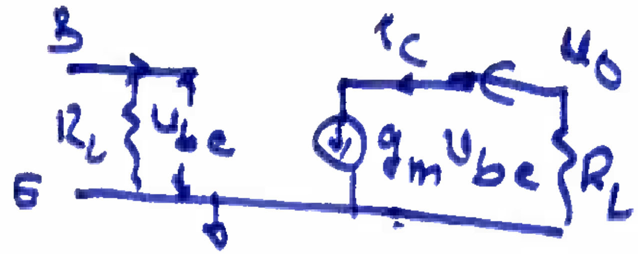
$$\Delta I_c = \frac{dI_c}{dV_{BE}} \cdot \Delta V_{BE}$$

OR

$$\Delta I_c = g_m \, \Delta V_{BE}$$

i.e.

$$i_c = g_m v_{be} \quad — (2)$$



$v_o = -i_c R_L$

$v_o = -\beta_o i_b R_L$

$i_b \cdot R_i = v_{be}$

$$\frac{v_o}{v_{be}} = -\frac{\beta_o R_L}{R_i}$$

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We have $I_C = I_S \exp(V_{BE}/V_T)$

$$\therefore \quad g_m = \frac{d}{dV_{BE}}\left[I_S \exp(V_{BE}/V_T)\right]$$

$$= \frac{I_S}{V_T}\,\mathrm{Exp}\,(V_{BE}/V_T) = \frac{I_C}{V_T} = \frac{I_C}{(kT/q)} = \frac{qI_C}{kT}$$

$$\therefore \quad g_m = \frac{qI_C}{kT} \quad \text{— } (3)$$

Typically at $27^\circ C$ (Room Temp.) $\frac{kT}{q} \simeq 26$ mV

$\therefore$ at $I_c = 1$ mA (say) $\quad g_m \cong 38$ mA/V

Small Signal Limitation

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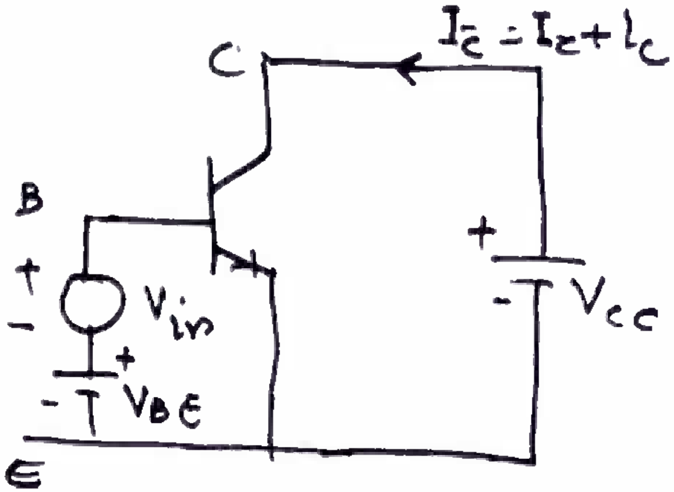
Example of CE Amplifier



$$I_{\bar{c}} = I_S \exp\left[\frac{V_{BE} + V_{in}}{V_T}\right] \quad \text{Total}$$

&

$$I_C = I_S \exp\left[\frac{V_{BE}}{V_T}\right] \quad \text{DC}$$

$$\therefore \; I_{\bar{c}} = I_C \exp\left(V_{in}/V_T\right)$$

If $V_{in} < V_T$

$$I_{\bar{c}} = I_C\left[1 + \frac{v_{in}}{V_T} + \frac{1}{2}\left(\frac{v_{in}}{V_T}\right)^2 + \frac{1}{6}\left(\frac{v_{in}}{V_T}\right)^3 + \cdots\right]$$

As $i_c = I_{\bar{c}} - I_C$

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Hence $i_c = \frac{I_c}{V_T} v_{in} + \frac{1}{2} \frac{I_c}{V_T^2} v_{in}^2 + \frac{1}{6} \frac{I_c}{V_T^3} v_{in}^3 \ldots\ldots$

If $v_{in} << V_T$ Then Higher Order terms are negligible

$\therefore\ i_c = \frac{I_c}{V_T} \cdot v_{in}$ & then $g_m = \frac{i_c}{v_{in}} = \frac{I_c}{V_T} = \frac{q I_c}{kT}$

Hence for Small Signal operation

$$\boxed{v_{in} << 26\,mV}$$

BJT Model (cont.): Capacitance

If a small signal voltage v_{in} is applied over dc value V_{BE}, then we say

$$\Delta V_{BE} = v_{in} \quad \& \quad \Delta Q_e = q_e$$

and $\Delta Q_b = q_b$

Then small signal (ac) capacitance across BE jn is given by

$$C_{be} = \frac{q_b}{v_{in}}$$

From Transistor theory, we know minority charge in Base is $Q_e = I_c \tau_B$ where τ_B is called Base Transit time

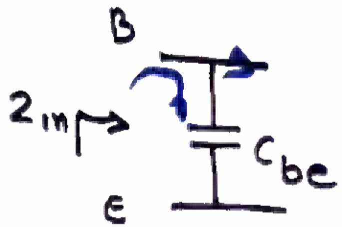
or $\Delta Q_e = \tau_B \Delta I_c$ From charge neutrality case $\Delta Q_e = \Delta Q_b$

$$\therefore \Delta Q_b = \Delta I_c \tau_B$$

or $q_b = i_c \tau_B$ (small signal representation)

Therefore $C_{be} = \dfrac{i_c \tau_B}{v_{in}} = \left(\dfrac{i_c}{v_{in}}\right)\tau_B = g_m \tau_B$

$$C_{be} = C_\pi' = \frac{q I_c}{kT} \cdot \tau_B \qquad \left(\tau_B = \frac{W_B^2}{2 D_n} \text{ for npn} \right)$$

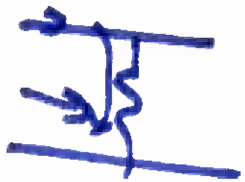


At Lower frequency $Z_{in} = \dfrac{1}{j\omega C_{be}}$ is large as C_{be} is normally very small (< pF range)

$\therefore$ At low Frequency, this shunting Capacitance can be neglected.

Input resistance

We have $I_C = \beta I_B$



or $I_B = \frac{1}{\beta} I_C$

or $\Delta I_B = \frac{d}{dI_C}\left(\frac{I_C}{\beta}\right)\Delta I_C$

We define ac β as $\beta_0 = \frac{\Delta I_C}{\Delta I_B} = \frac{i_c}{i_b} = \left[\frac{d}{dI_C}\left(\frac{I_C}{\beta}\right)\right]^{-1}$

β_0 is called Small Signal Current Gain

If β is constant, $\beta_0 = \beta_{DC}$ else β_0 is different from β.

Then, Input Resistance $r_\pi = \frac{v_{in}}{i_b} = \frac{v_{in}}{i_c} \cdot \beta_0 = \frac{\beta_0}{g_m}$

$\therefore \boxed{\beta_0 = g_m r_\pi}$ or $\frac{i_c}{i_b} = \left(\frac{kT}{qI_C}\right)^{-1} \cdot r_\pi = \frac{I_C}{I_B}$ $\therefore \boxed{r_\pi = \frac{kT}{qI_B}}$

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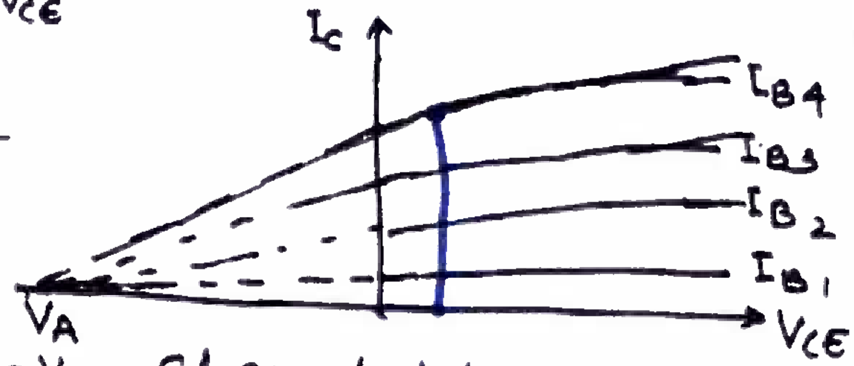
Output Resistance

We have found earlier

$$\Delta I_c = \frac{\Delta I_c}{\Delta V_{CE}} \Delta V_{CE}$$

or $$\frac{\Delta V_{CE}}{\Delta I_c} = \frac{V_A}{I_c}$$



$\frac{\Delta V_{CE}}{\Delta I_c}$ is slope of I_c-V_{CE} characteristics

$\therefore$ Output Resistance $r_o = \frac{\Delta V_{CE}}{\Delta I_c} = \frac{v_{ce}}{i_c} = \frac{V_A}{I_c}$

where V_A is Early Voltage

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We define a parameter η is

$$\eta = \frac{kT}{q} \cdot \frac{1}{V_A}$$

$$= \frac{kT}{qI_c} \cdot \frac{I_c}{V_A} = \frac{1}{g_m} \cdot \frac{1}{r_o}$$

or $$\boxed{g_m r_o = \frac{1}{\eta}}$$

Typically V_A varies between 50 to 100 Volts

Further at $T = 300^\circ k$, $\eta \simeq 2.6 \times 10^{-4}$ for above $V_A = 100V$

For operating current of $I_c = 1mA$, we get $r_o = \frac{100}{10^{-3}} = 100k\Omega$

Base-Emitter J^n capacitance C_{je}

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As we did evaluation for C_μ, we do the same for C_{je} which is BE J^n capacitance

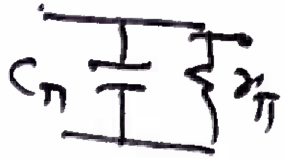
$$C_{je} = \frac{C_{jeo}}{\left(1 - \frac{V_{BE}}{\phi_0}\right)^n}$$

. But this is not value which is Valid.

BE J^n is FB and hence this formulae is not correct

Typically we take $\boxed{C_{je} = 2 \cdot C_{jeo}}$

Then Input capacitance $\boxed{C_\pi = C_{be} + C_{je}}$



Base-Collector J^n capacitance C_μ

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C_μ => Varies with variation in V_{CB} as junction is in Reversed-Biased State.

We know RB J^n capacitance of a diode (with Step J^n) is given by

$$C_j = \frac{C_{j0}}{\left(1 - \frac{V}{\phi_0}\right)^{1/2}}$$

(1/2 may become 1/3 for Linearly Graded junction)

ϕ_0 = Built-in Potential $= \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$

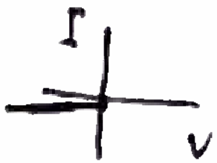
Hence in Transistor

$$C_\mu = C_{\mu 0} / \left(1 - \frac{V_{CB}}{\phi_0}\right)^n \quad (n = 1/2 \text{ or } 1/3)$$

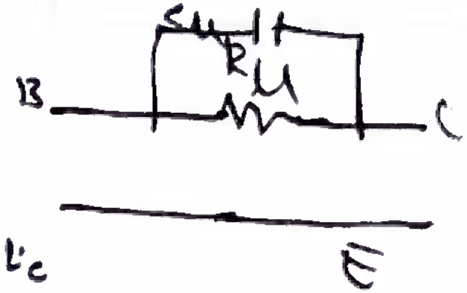
Collector - Base resistance r_μ

As V_{BE} gets modulated due to V_{in}, the Base-Collector junction bias too get modulated. That V_{CE} changes due to change in I_B

We define



$$r_\mu = \frac{\Delta V_{CE}}{\Delta I_B} = \frac{\Delta V_{CE}}{\Delta I_C} \cdot \frac{\Delta I_C}{\Delta I_B} = \frac{v_{ce}}{i_c} \cdot \frac{i_c}{i_b}$$



$\therefore$ $\boxed{r_\mu = r_o \cdot \beta_o}$ This resistance is between ~~Base J'~~ Base and Collector regions.

Collector Substrate Capacitance C_{cs}

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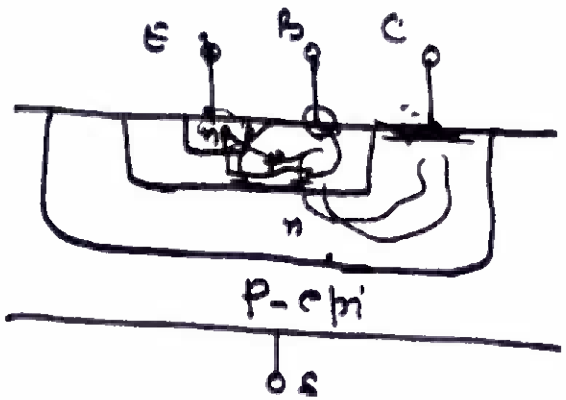
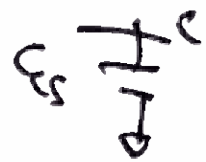


Since in an IC, collector is a diffused region in substrate, one observes presence of a collector - substrate junction with capacitance

$$C_{js} = \frac{C_{cso}}{\left(1 - \frac{V_{cs}}{\phi_0'}\right)^n}$$



Normally substrate is grounded. Hence C_{cs} is the collector capacitance between collector terminal & Ground

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Parasitic Resistances $r_{bb'}$, r_{es} & r_c

1. $r_{bb'}$:- Base Resistance

2. r_{es} :- Emitter Region Resistance

3. r_c :- Collector Region Resistance.

Using these parameters we can now create the Small Signal Equivalent Circuit of a Bipolar Transistor.

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Equivalent Circuit of BJT

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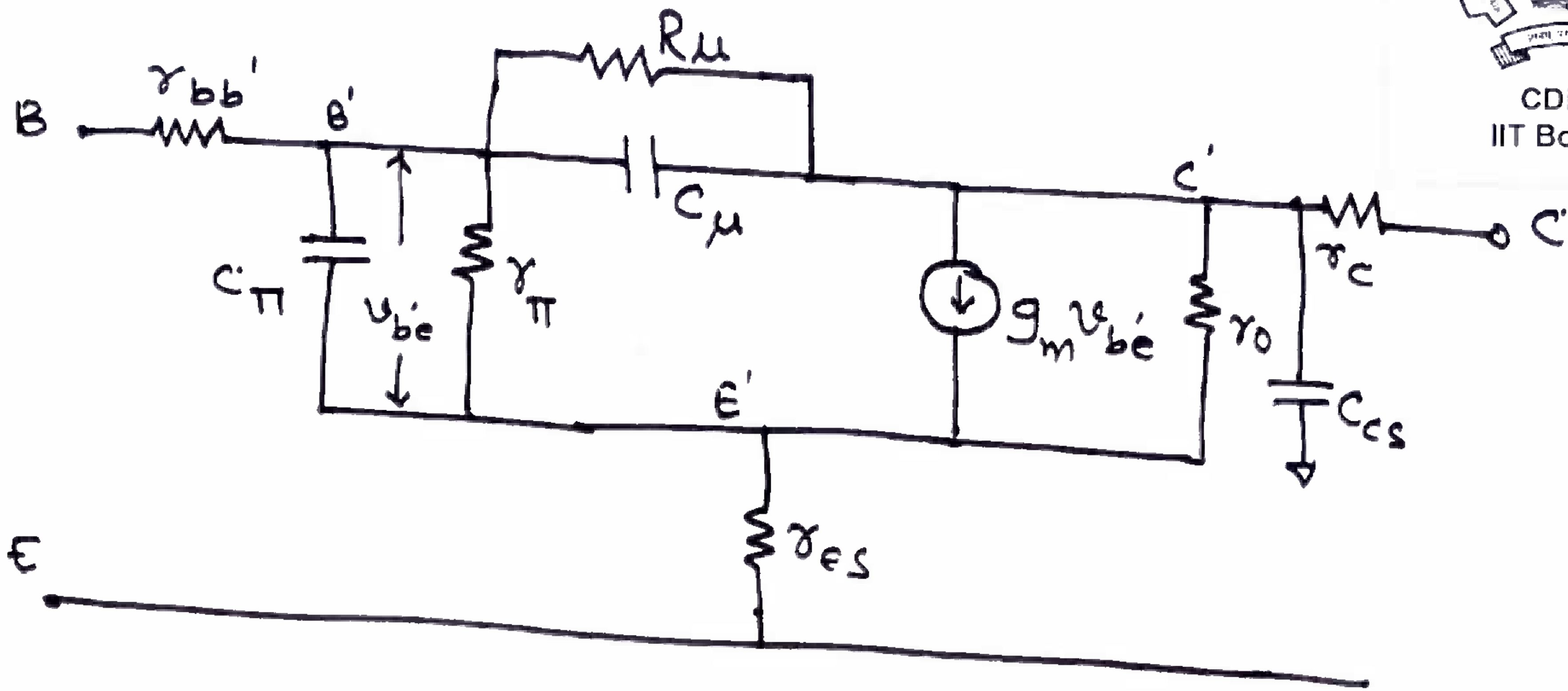

Course Name
Analog Circuits

Lecture No.

Instructor's Name
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