Bipolar Transistor Models

\[ I_E = I_B + I_C \quad (Universal) \quad - (1) \]

\[ I_C = \alpha I_E \quad \alpha = \alpha_T \gamma \quad - (2) \]

\[ \frac{I_C}{I_B} = \beta \quad . \quad \beta = \frac{\alpha}{1 - \alpha} \quad - (3) \]

\( \beta \) is called Forward Current Gain

\[ I_C = \alpha I_E + I_{CO} \quad - (4) \]

\[ I_{CO} = I_{CS} (1 - \alpha_F \alpha_R) \quad - (5) \]

\[ V_{CE} = V_{BE} + V_{BC} \quad - (6) \]
Small-Signal Model of BJT

\[ I_b = i_b + I_B \]

Total = Small Signal + DC

\[ I_C = i_C + I_C \]

\[ v_{be} = V_{be} + v_{be} \]
Small Signal Model (Cont.)

Most important parameter of Transistor is Transconductance

\[ g_m = \frac{\partial I_C}{\partial V_{BE}} \]  \hspace{1cm} (1)

\[ I_C = I_s \exp \left( \frac{V_{BE}}{V_T} \right) \]

We have

\[ \Delta I_C = \frac{dI_C}{dV_{BE}} \cdot \Delta V_{BE} \]

or

\[ \Delta I_C = g_m \Delta V_{BE} \]

i.e.

\[ i_C = g_m v_{be} \]  \hspace{1cm} (2)

\[ \frac{v_o}{v_{be}} = -\beta_0 \frac{R_L}{R_i} \]

\[ v_o = -i_C R_L \]

\[ v_o = -\beta_0 i_b R_L \]
We have \( I_C = I_S \exp \left( \frac{V_{BE}}{V_T} \right) \)

\[ g_m = \frac{d}{dV_{BE}} \left[ I_S \exp \left( \frac{V_{BE}}{V_T} \right) \right] \]

\[ = \frac{I_S}{V_T} \exp \left( \frac{V_{BE}}{V_T} \right) = \frac{I_C}{V_T} = \frac{I_C}{(kT/q)} = \frac{9I_C}{kT} \]

\[ g_m = \frac{9I_C}{kT} \quad - \quad (3) \]

Typically at 25°C (Room Temp.) \( \frac{kT}{q} \approx 26 \text{ mV} \)

\[ \text{at } I_C = 1 \text{ mA (say)} \quad g_m \approx 38 \text{ mA/V} \]
Small Signal Limitation

Example of CE Amplifier

\[ I_C = I_e + I_C \]

\[ I_C = I_S \exp \left[ \frac{V_{BE} + V_{in}}{V_T} \right] \text{ Total} \]

\[ I_C = I_S \exp \left[ \frac{V_{BE}}{V_T} \right] \text{ DC} \]

\[ I_C = I_C \exp \left( \frac{V_{in}}{V_T} \right) \]

If \( V_{in} < V_T \)

\[ I_C = I_C \left[ 1 + \frac{V_{in}}{V_T} + \frac{1}{2} \left( \frac{V_{in}}{V_T} \right)^2 + \frac{1}{6} \left( \frac{V_{in}}{V_T} \right)^3 + \cdots \right] \]

As \( i_C = I_C - I_e \)
Hence 
\[ i_c = \frac{I_c}{V_T} \cdot V_{in} + \frac{1}{2} \frac{I_c}{V_T^2} \cdot V_{in}^2 + \frac{1}{6} \frac{I_c}{V_T^3} \cdot V_{in}^3 \ldots \]

If \( V_{in} \ll V_T \) Then Higher Order terms are negligible

\[ i_c = \frac{I_c}{V_T} \cdot V_{in} \]

Then \( g_m = \frac{i_c}{V_{in}} = \frac{I_c}{V_T} = \frac{2I_c}{kT} \)

Hence for Small Signal operation

\[ V_{in} \ll 26 \text{mV} \]
BJT Model (cont.): Capacitance

If a small signal voltage \( V_{in} \) is applied over dc value \( V_{BE} \), then we say

\[
\Delta V_{BE} = V_{in} \quad \Delta Q_e = q_e
\]

and \( \Delta Q_b = q_b \)

Then small signal (ac) capacitance across BE junction is given by

\[
C_{be} = \frac{q_b}{V_{in}}
\]

From transistor theory, we know minority charge in Base is \( Q_e = I_c \tau_B \), where \( \tau_B \) is called Base Transit time

or \( \Delta Q_e = \tau_B \Delta I_c \) from charge neutrality \( \cos \Delta Q_e = \Delta Q_b \)
\[ \Delta Q_b = \Delta I_c \tau_B \]

\[ q_b = I_c \tau_B \quad \text{(small signal representation)} \]

Therefore \[ C_{be} = \frac{I_c \tau_B}{V_{in}} = \left( \frac{I_c}{V_{in}} \right) \tau_B = q_m \tau_B \]

\[ C_{be} = C_{n} = \frac{q I_c}{kT} \cdot \tau_B \quad \left( \tau_B = \frac{W^2}{2 \cdot Dn} \text{ for npn} \right) \]

At lower frequency, \[ Z_{in} = \frac{1}{j\omega C_{be}} \] is large as \( C_{be} \) is normally very small (\(<\)pf range)

\[ \therefore \text{At low frequency, this shunting capacitance can be neglected.} \]
**Input Resistance**

We have \( I_c = \beta I_B \)

\[
\alpha \quad I_B = \frac{1}{\beta} I_c
\]

\[
\alpha \quad \Delta I_B = \frac{d}{dI_c} \left( \frac{I_c}{\beta} \right) \Delta I_c
\]

We define \( \beta_c \) as \( \beta_c = \frac{\Delta I_c}{\Delta I_B} = \frac{I_c}{I_B} = \left[ \frac{d}{dI_c} \left( \frac{I_c}{\beta} \right) \right]^{-1} \)

\( \beta_c \) is called Small Signal Current Gain

If \( \beta \) is constant, \( \beta_c = \beta \) else \( \beta_c \) is different from \( \beta \).

Then, Input Resistance \( R_I = \frac{V_{in}}{I_B} = \frac{V_{in}}{I_c} \cdot \beta_c = \frac{V_{in}}{I_c} \cdot \beta_c \)

\[
\therefore \quad \beta_c = \frac{R_I}{R_I} \quad \alpha \quad \frac{I_c}{I_B} = \left( \frac{kT}{qI_c} \right)^{-1} \quad R_I = \frac{I_c}{I_B} \quad \therefore \quad R_I = \frac{kT}{qI_c}
\]
Output Resistance

We have found earlier

$$\Delta I_c = \frac{\Delta I_c}{\Delta V_{ce}} \Delta V_{ce}$$

$$\frac{\Delta V_{ce}}{\Delta I_c} = \frac{V_A}{I_c}$$

$$\frac{\Delta V_{ce}}{\Delta I_c}$$ is slope of $I_c-V_{ce}$ characteristics

$$\therefore \text{Output Resistance } r_o = \frac{\Delta V_{ce}}{\Delta I_c} = \frac{V_{ce}}{I_c} = \frac{V_A}{I_c}$$

where $V_A$ is Early Voltage
We define a parameter $\eta$ as

$$\eta = \frac{kT}{q} \cdot \frac{1}{V_A}$$

$$= \frac{kT}{qI_C} \cdot \frac{I_C}{V_A} = \frac{1}{q m} \frac{1}{\gamma_0}$$

$$\alpha \gamma_m \gamma_0 = \frac{1}{\eta}$$

Typically $V_A$ varies between 50 to 100 Volts.

Further at $T=300^\circ K$, $\gamma \approx 2.6 \times 10^4$ for above $V_A = 100V$.

For operating current of $I_C = 1mA$, we get $\gamma_0 = \frac{100}{10^3} = 100 \text{pV}$. 

Course Name: Analog Circuits
Lecture No.: 14
Instructor's Name: Prof. A. N. Chandorkar
Base-Emitter Jn capacitance \( C_{je} \)

As we did evaluation for \( C_n \), we do the same for \( C_{je} \) which is BE Jn capacitance

\[
C_{je} = \frac{C_{jeo}}{(1 - \frac{V_{BE}}{V_0})^m}
\]

But this is not value which is Valid.

BE Jn is FB and hence this formulae is not correct

Typically we take \( C_{je} = 2 \cdot C_{jeo} \)

Then Input capacitance \( C_{ni} = C_{be} + C_{je} \)
Base-Collector Jn Capacitance $C_{j\mu}$

$C_{\mu} \Rightarrow \text{varies with variation in } V_{CB} \text{ as junction is in Reversed-Biased State.}$

We know RB Jn capacitance of a diode (with Step Jn) is given by

$$C_j = \frac{C_{j0}}{(1 - \frac{V}{\Phi_0})^{1/2}}$$

($\frac{1}{2} \text{ may become } \frac{1}{3}$

for linearly graded junction)

$\Phi_0 = \text{Built-in Potential} = \frac{kT}{e} \ln \frac{N_a N_D}{n_i^2}$

Hence in Transistor

$$C_{\mu} = \frac{C_{\mu 0}}{(1 - \frac{V_{CB}}{\Phi_0})^n} \quad (n = 1/2 \text{ or } 1/3)$$
Collector-Base resistance $\gamma_\mu$

As $V_{BE}$ gets modulated due to $V_{in}$, the Base-Collector junction bias too get modulated that $V_{CE}$ changes due to change in $I_B$

We define

$$\gamma_\mu = \frac{\Delta V_{CE}}{\Delta I_B} = \frac{\Delta V_{CE}}{\Delta I_c} \cdot \frac{\Delta I_c}{\Delta I_B} = \frac{V_{CE}}{I_c} \cdot \frac{I_c}{I_B}$$

$$\therefore \gamma_\mu = \gamma_0 \cdot \beta_0$$

This resistance is between Base and Collector regions.
Collector Substrate Capacitance $C_{cs}$

Since in an IC, collector is a diffused region in substrate, one observes presence of a Collector-Substrate junction with capacitance

$$C_{js} = \frac{C_{cs0}}{(1 - \frac{V_{cs}}{\Phi_0})^n}$$

Normally Substrate is Grounded. Hence $C_{cs}$ is the collector capacitance between Collector terminal & Ground.
Porous Resistances $r_{bb'}, r_{ed} r_c$

1. $r_{bb'}$ - Base Resistance

2. $r_{es}$ - Emitter Region Resistance

3. $r_c$ - Collector Region Resistance.

Using these parameters we can now create the Small Signal Equivalent Circuit of a Bipolar Transistor.
Equivalent Circuit of BJT