Frequency Response of Amplifier

S-Domain Analysis

We know

\[ j\omega = s \]

Any Transfer Function of a Network is given as

\[
H(s) = H(j\omega) = \frac{V_o(s)}{V_i(s)}
\]

\[
= K \frac{(s-z_1)(s-z_2) \cdots (s-z_m)}{(s-p_1)(s-p_2) \cdots (s-p_n)}
\]

where \( z \) is called 'zero' of the transfer function

\( p \) is called 'pole' of the transfer function
\[ \frac{V_o(s)}{V_{in}(s)} = \frac{R_L C \cdot s}{(R_L+R_S)C \cdot s + 1} = H(s) \]

\[ i(s) = \frac{V_{in}}{R_s + \frac{1}{C s} + R_L} = \frac{V_{in} C s}{(R_s+R_L)C s + 1} \]

\[ V_o(s) = i(s) \cdot R_L \]
(1) If \( s = 0 \) then \( H(s) \to 0 \)

\[ \therefore \text{we have a 'Zero' at } \omega = 0 \]

(2) If \( (R_L + R_S)C \cdot s = -1 \)

Then \( \lim_{s \to \infty} H(s) \to \infty \)

\[ \therefore \text{at } s = \frac{-1}{(R_L + R_S)C} \text{ we have a Pole,} \]

We see that RC has unit of Time

\[ \therefore T_S = (R_L + R_S)C \]
We can rewrite $H(s)$ as

$$H(s) = \frac{U_{o}(s)}{U_{i}(s)} = \frac{R_L}{R_L + R_S} \cdot \frac{(R_L + R_S)c_s}{1 + (R_L + R_S)c_s}$$

$$= K \cdot \frac{s \tau_S}{1 + s \tau_S}$$

$$\Rightarrow Av(s) = \frac{R_L}{R_L + R_S} \cdot \frac{j \omega \tau_S}{1 + j \omega \tau_S}$$

1. $\tau_S = \frac{R_L}{R_L + R_S}$
2. $\omega \tau_S \rightarrow 2$
3. $\frac{1}{1 + j \omega \tau_S} \rightarrow 3$
\[ \left| A_u(j\omega) \right| = \frac{R_L}{R_L + R_s} \left[ \frac{\omega T_s}{\sqrt{1 + \omega^2 T_s^2}} \right] \]

Where \( \omega = 2\pi f \)

\[ \left| A_u(j\omega) \right| = \frac{R_L}{R_L + R_s} \left[ \frac{2\pi f T_s}{1 + 4\pi^2 f^2 T_s^2} \right]^{1/2} \]

In dBs,

\[ \left| A_u(j\omega) \right|_{\text{db}} = 20 \log_{10} \left( \left| A_u(j\omega) \right| \right) \]

\[ P_{\text{dB}} = 20 \log \frac{P_2}{P_1} \]

\[ A_{\text{dB}} = 20 \log \frac{V_2}{V_1} \]
\[ |A_v(f)|_{\text{dB}} = 20 \log \left( \frac{R_L}{R_L + R_s} \right) \]

\[ + 20 \log \left( 2\pi f \tau_s \right) \]

\[ - 20 \log \left[ \sqrt{1 + (2\pi f \tau_s)^2} \right] \]

**1st Term**

\[ 20 \log \left( \frac{R_L}{R_L + R_s} \right) \]

\[ + \text{db} \]

\[ - \text{ve} \]

\[ \omega(f) \]
And Term

\[ f = \frac{1}{2\pi f_s} \]

Slope 20\,\text{db/decade}

\[ f(\text{log scale}) \]

And Term

\[ f = \frac{1}{2\pi f_s} \]

Slope 20\,\text{db/decade}

\[ -20\,\text{log} (\sqrt{2}) \]
Complete function $A_v(j\omega)$'s frequency response can then be plotted as

$$20 \log |A_v(j\omega)|$$

with an asymptotic curve and a real curve. The corner frequency is $f = \frac{1}{2\pi c}$. The Bode plot is shown with the asymptotic and real curve superimposed.
A complex term = \( x + jy = f \)

Then \( |f| = \sqrt{x^2 + y^2} \) - Magnitude

\( \theta = \angle f(jw) = \tan^{-1} \frac{y}{x} \) - Phase.

![Diagram showing real and imaginary parts of a complex number]
\[ f = x + jy \]

**Phase part**

\[ \tan^{-1} \frac{y}{x} = \theta \]

If \( y = x \) \( \tan^{-1} \) then \( \theta = 45^\circ \)

If \( y = 0 \) \( \tan^{-0} \) then \( \theta = 0^\circ \)

If \( x = 0 \) \( \tan^{-\infty} \) then \( \theta = 90^\circ \)

\( x \rightarrow \infty \)

Take a case of Frequency response

\[ \theta = \tan^{-1} \frac{w}{w_0} \]

At \( w = w_0 \) \( \theta = 45^\circ \)

At \( w = 0.1w_0 \) \( \theta = \tan^{-1}(0.1) \rightarrow 0^\circ \)

At \( w = 10w_0 \) \( \tan^{-1} 10 = \theta \rightarrow 90^\circ \)

i.e. \( \theta \) changes \( 45^\circ /\text{decade} \)
Phase Response of $A_\nu(j\omega)$

\begin{align*}
90^\circ & \quad f = \frac{1}{2\pi \nu_0} \\
45^\circ & \quad f = \frac{1}{4\pi \nu_0} \\
0^\circ & \quad f = \frac{1}{2\pi \nu_0}
\end{align*}
Two Pole Circuit

\[ V_o \]

\[ \frac{V_o(s)}{V_{in}(s)} = A_v(s) = \frac{R_p}{R_p + R_s} \cdot \frac{1}{[1 + \frac{R_p}{R_p + R_s} \left( \frac{C_p}{C_s} \right) + \frac{1}{sT_s} + sT_p]} \]

where

\[ T_s = (R_s + R_p) \cdot C_s \]

\[ T_p = (R_s \parallel R_p) \cdot C_p \]

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$\tau_s$ is called open circuit time constant

1. c. when $\frac{1}{C_p \omega} \rightarrow 0$

$\therefore \ \tau_s$ represents time constant associated with $C_s$

Similarly $\tau_p$ is called short circuit time constant

2. c. when $\frac{1}{C_s S} \rightarrow 0$

$\therefore \ \tau_p$ is time constant associated with $C_p$
Two poles are associated with these Two Time constants. We shall see later that

\[ f_L = \frac{1}{2\pi \tau_L} \] is called Lower Corner (3dB) Frequency

and

\[ f_H = \frac{1}{2\pi \tau_P} \] and is called Upper Corner (3dB) Frequency.

\[ \left| \frac{V_o(s)}{V_{in}} \right| \]

\[ f_L \quad f_H \]

\[ f_H - f_L \] is called Midband and is essentially Bandwidth.
Miller's Theorem

\[ V_1 \rightarrow -AV_1 \quad \text{output} = \quad (1+A)Z \]

Two Port Network

Condition for Validity of Miller's Theorem:

We must have Two Paths from Input to Output.
Concept of Pole-Zero (Revisited)

A typical amplifier is shown here.

If $C_\text{in}$ is input capacitance at Node 1, then $C_\text{in}$ must contain contribution from $C_{\text{in}_1}$ and $C_c$.

$C_c$ is of course capacitance at node 1 and node 2 too.

We can say pole's association is with Time Constant.

\[ P_i = \frac{1}{R_c C_{\text{in}}} \]
For our Amplifier, the Gain

\[ A_u = -g_m R_D \]

\[ &= \text{If } C_e \text{ acts like component (impedance) between Input & Output, we can convert it's contribution on Input & Output side independently using Milley's theorem} \]
On Input Side

\[ I \left( 1 - \left( -g_m R_D \right) \right) C_c \]

or

\[ C_{in} = C_{in1} + \left( 1 + g_m R_D \right) C_c \]

\[ = C_{in1} + (g_m R_D) C_c = C_{in1} + |A_v| C_c \]

On the Output side then

\[ P_1 = \frac{1}{R_s \cdot C_{in}} \quad (\text{KHz}) \]

\[ C_c \left( \frac{A}{1+A} \right) = C_c \]

\[ \therefore \text{Pole}_2 = P_2 = \frac{1}{R_D (C_{out} = \frac{1}{R_D (C_{L+Cc})} \]

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All Pass

HP Filter

Low Pass Filter

Notch Filter

Band Reject
\[ \frac{1}{S^2 + \alpha S + \delta} = \frac{1}{(S + \omega_1)(S + \omega_2)} = \frac{1}{\omega_2 \left( \frac{\omega_2}{\omega_1} + 1 \right)} \]

Stability
Concept of Zero

If magnitude of two signals are equal, then net $V_0 = 0$ due to 180° phase shift.

This is called Zero

$Z_1 = \frac{9m}{C_0}$

It can be shown that $V_0 = 0$ due to 180° phase shift.