

Small Signal Low Frequency MOS MODEL

Small Signal means amplitude of signal is very much smaller than (amplitude) or value of DC bias. \checkmark

i.e.
$$v_{GS} = V_{GS} + v_{gs}$$

$$v_{DS} = V_{DS} + v_{ds}$$

$$i_{DS} = i_{ds} + I_{DS}$$

For a MOSFET with small signal input at Gate (V_{GS}) we have

$$i_{DS} = I_{DS} + i_{ds} = \frac{\beta}{2} [(V_{GS} + v_{gs}) - V_{TH}]^2 (1 + \lambda v_{DS})$$



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Assuming $\lambda \rightarrow 0$

$$i_{DS} = \frac{\beta}{2} (V_{GS} - V_T)^2 + \frac{\beta}{2} (v_{gs})^2 + 2 \cdot \frac{\beta}{2} (V_{GS} - V_T) v_{gs}$$

For small signal case $v_{gs}^2 \rightarrow$ Small \rightarrow Neglected

$$\therefore i_{DS} = I_{DS} + i_{ds}$$

$$= \frac{\beta}{2} (V_{GS} - V_T)^2 + \beta (V_{GS} - V_T) v_{gs}$$

$$\therefore i_{ds} = \beta (V_{GS} - V_T) v_{gs}$$

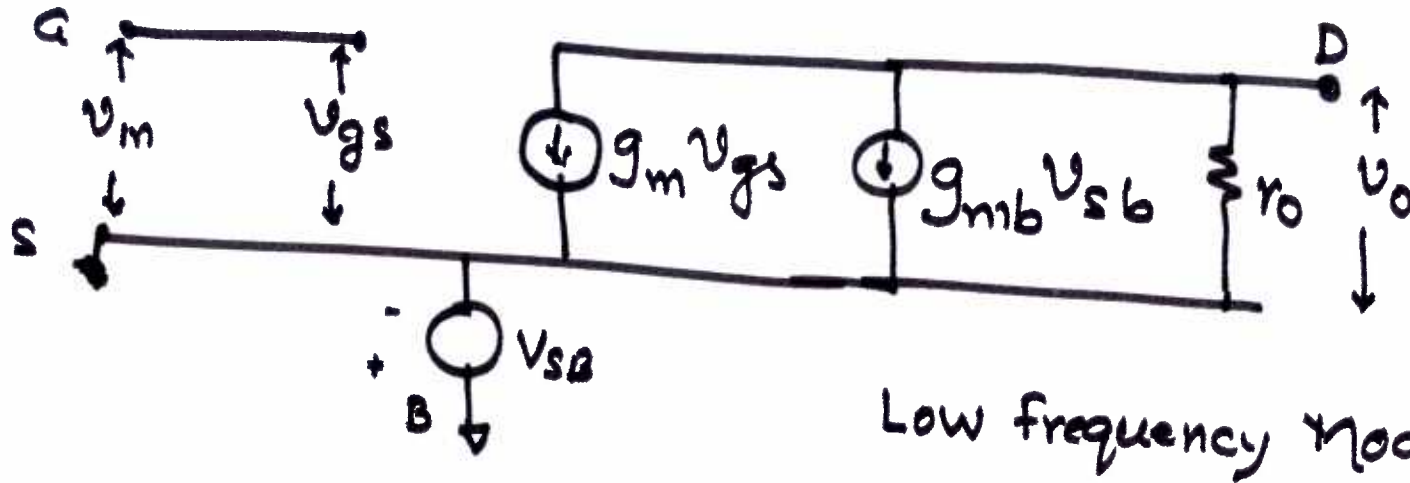
$$\frac{i_{ds}}{v_{gs}} = g_m = \beta (V_{GS} - V_T) = \frac{1}{2} \beta \frac{(V_{GS} - V_T)^2 \times 2}{(V_{GS} - V_T)} = \sqrt{2\beta I_{DS}}$$



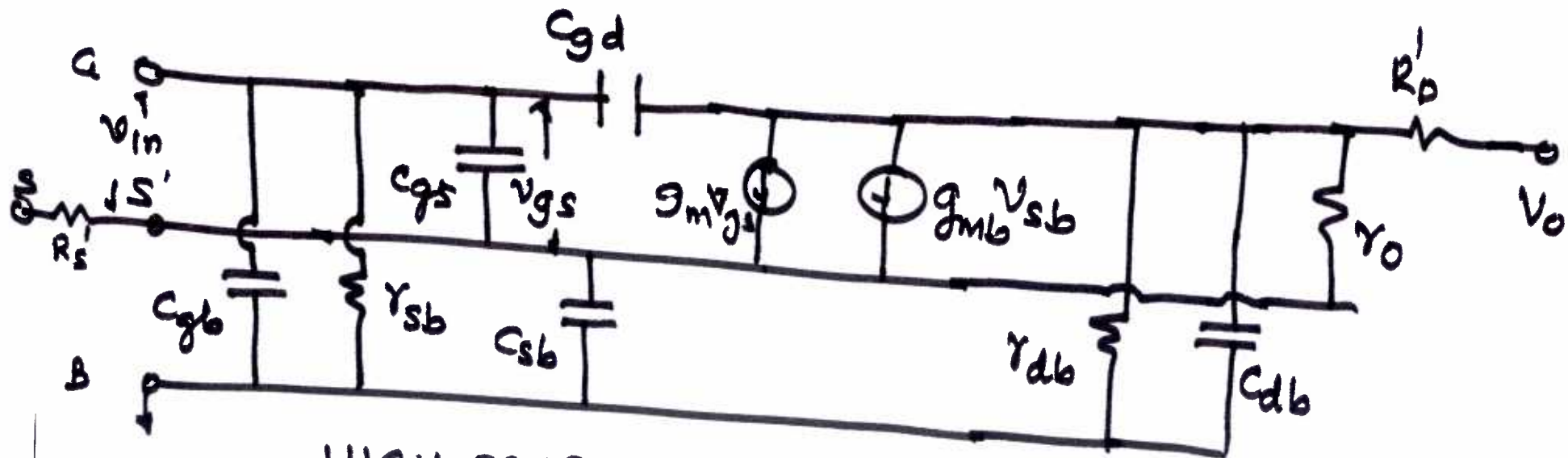
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Small Signal MODEL



Low frequency Model



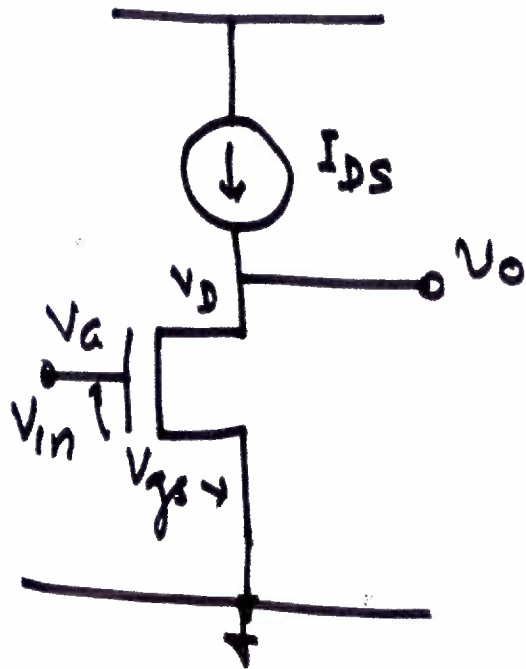
HIGH FREQUENCY SMALL SIGNAL MOS MODEL



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Common Source Amplifier in an IC

Amplifier with constant current biasing (I_{DS} const.)



We have $I_{DS} = f(V_G \& V_D)$

$$\therefore \Delta I_{DS} = \frac{\partial I_{DS}}{\partial V_G} \cdot \Delta V_G + \frac{\partial I_{DS}}{\partial V_D} \cdot \Delta V_D$$

By definition

$$g_m = \frac{\partial I_{DS}}{\partial V_G} \quad \& \quad g_o = \frac{\partial I_{DS}}{\partial V_D}$$

$$\therefore \Delta I_{DS} = g_m \Delta V_G + g_o \Delta V_D$$

$$\text{or } dI_{DS} = g_m dV_G + g_o dV_D$$

$$\text{But } v_{in} = dV_G \quad \text{and} \quad v_o = dV_D$$



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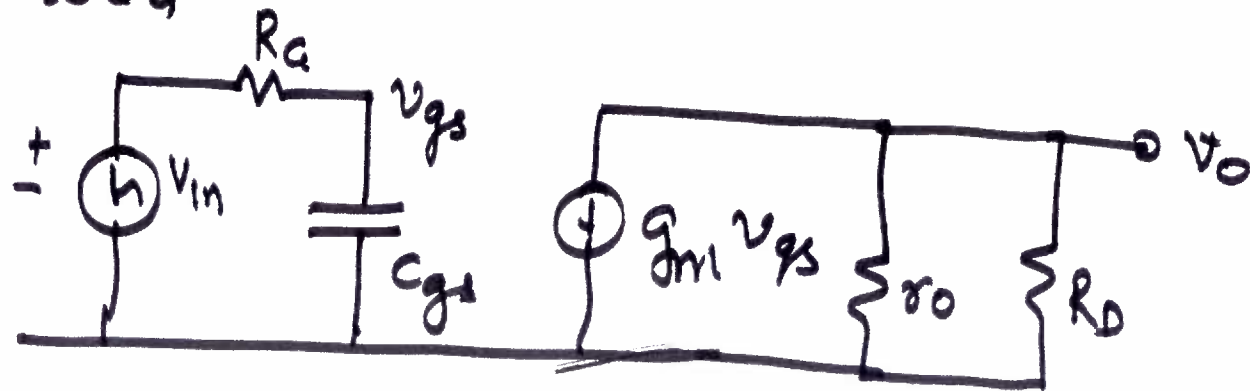
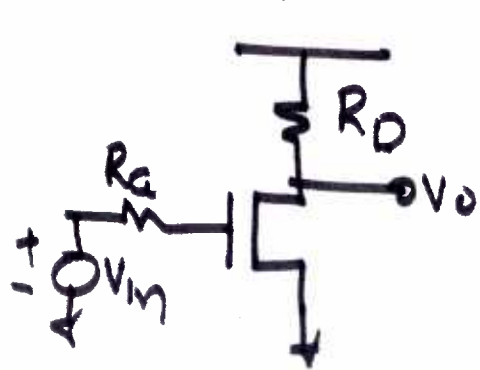
$$\therefore dI_{DS} = g_m v_{in} + g_o v_o$$

As Amplifier is biased by Constant Current Source I_{DS} , Hence $dI_{DS} = 0$

$$\therefore 0 = g_m v_{in} + g_o v_o$$

$$\therefore \frac{v_o}{v_{in}} = - \frac{g_m}{g_o} = - g_m r_o$$

If we take First Order Model of MOSFET with Resistive load



If $r_{od} = r_o$

Then $A_{vo} = -g_m r_o$ or $\left| \frac{r_o}{A_{vo}} \right| = \frac{1}{g_m}$

We can see that

$$\frac{g_m}{I_{D_s}} = \frac{2}{V_{ov}} \quad \& \quad \frac{g_m}{C_{gs}} = \frac{3}{2} \mu \frac{V_{ov}}{L^2}$$

can be termed as Figures of Merit

Hence Major decision for any Analog Designer is to choose V_{ov} appropriately, so that

1. Power Dissipation
 2. Gain
 3. Bandwidth
- specs are met



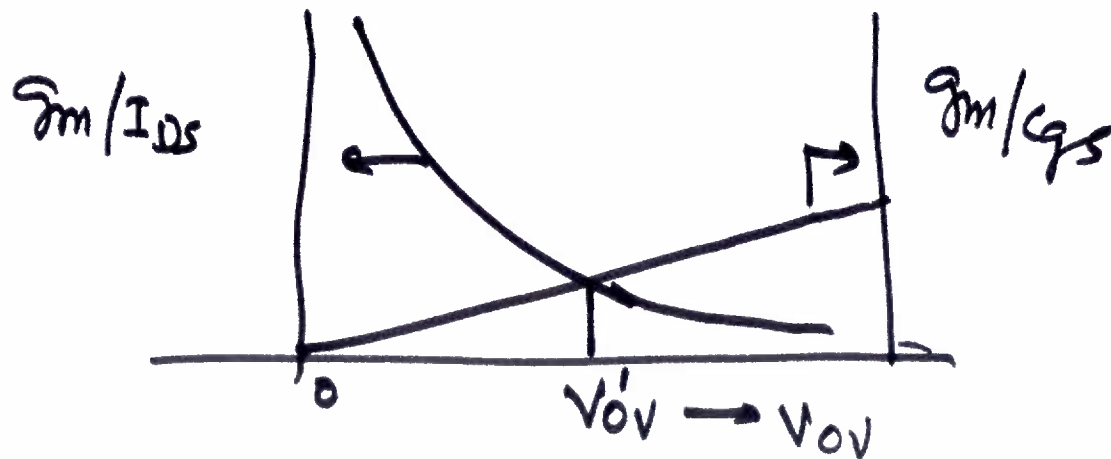
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We have observed that Performance of MOSFET ^{Amplifier} can be defined by two Figure of Merit, namely

where $\frac{g_m}{I_{D_S}}$ and $\frac{g_m}{C_{gs}}$

$$\frac{g_m}{I_{D_S}} = \frac{2}{V_{ov}} \quad \text{and} \quad \frac{g_m}{C_{gs}} = \frac{3}{2} \left(\frac{\mu}{L^2} \right) V_{ov}$$



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Optimum value of $V_{ov} = V_{ov}'$ from Figure.

Hence Designer should make reasonable correct choice of V_{ov} .

Another Figure of Merit could be defined as Product-

$$FM_3 = \frac{g_m}{I_{DS}} \cdot \frac{g_m}{C_{gs}} = \frac{2}{V_{ov}} \cdot \frac{3}{2} \left(\frac{\mu}{L^2} \right) V_{ov}$$

$$= 3 \left(\frac{\mu}{L^2} \right)$$

Which is not a function of V_{ov} .

Hence to improve FM_3 , we must reduce Channel Length L .

FM_3 essentially represents
Think How about A_v ?!

SPEED & POWER Efficiency together.



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Then

$$H(s) = \frac{v_o(s)}{v_{in}(s)} = -g_m (r_o \parallel R_D) \cdot \frac{1}{1 + s R_a C_{gs}}$$

We can use $g_m = \frac{2I_{DS}}{V_{OV}}$

$$C_{gs} = \frac{2}{3} W \cdot L C_{ox} \quad (\sim C_{ox})$$

$$r_o = \frac{1}{\lambda I_{DS}}$$

Then $A_v(s) = H(s) = A_{VO} \frac{1}{1 + s(R_g C_{gs})}$

where $A_{VO} = -g_m (r_o \parallel R_D) = -g_m R_D$ if $R_D \ll r_o$
 $= g_m r_o$ if $R_D \gg r_o$

↳ say from Current source



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From $A_v(s)$, we see we have a pole
at $\omega_0 = \frac{1}{R_G C_{gs}}$, which is therefore
the Bandwidth.

If we write $C_{gs} = \frac{2}{3} W L C_{ox}$

But $C_{ox} \mu \frac{W}{L} V_{ov} = g_m$

$$\therefore C_{ox} = g_m \left(\frac{L}{W}\right) (V_{ov})^{-1} \frac{1}{\mu}$$

$$C_{ox} = \frac{A_{vo}}{(r_{o1} || R_D)} \cdot \frac{L}{W} \cdot \frac{1}{V_{ov}} \cdot \frac{1}{\mu}$$

$$\therefore C_{gs} = \frac{2}{3} W L \cdot \frac{A_{vo}}{r_{od}} \left(\frac{L}{W}\right) \frac{1}{V_{ov}} \cdot \frac{1}{\mu}$$

$$\therefore \omega_0 = \frac{3}{2} \frac{r_{od}}{R_G} \cdot \frac{1}{A_{vo}} \frac{\mu}{L^2} V_{ov}$$

We define
 $r_{od} = r_{o1} || R_D$