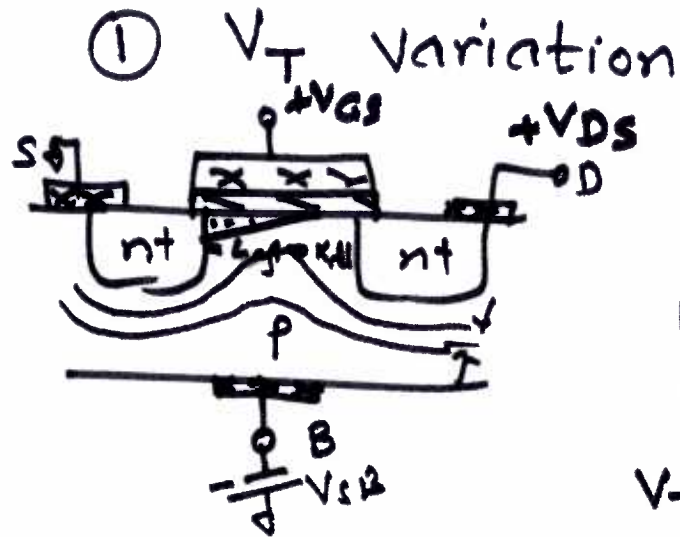


# Analog MOS Model



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As  $V_{SB}$  increases(-), Depletion

Layer increases, which in turn increases  $Q_B$ . Hence  $V_T$  increases.

$V_T$  with  $V_{SB}$  is given by

$$V_T = V_{T0} + \gamma \left[ (V_{SB} + 2\phi_F)^{1/2} - (2\phi_F)^{1/2} \right]$$

where  $V_T$  is Threshold Voltage with  $V_{SB} \rightarrow -ve$   
 $V_{T0}$  is Threshold Voltage with  $V_{SB} = 0$

$$\gamma = \frac{1}{C_{ox}} \sqrt{2K_s \epsilon_0 q N_{a,d}}$$

Let us find variation of  $V_T$  with  $V_{SB}$

$$\frac{\partial V_T}{\partial V_{SB}} = 0 + \frac{\gamma}{2} (V_{SB} + 2\phi_F)^{-1/2}$$



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② The saturation parameter  $\lambda$ , has been described as  $\frac{\lambda'}{L}$  and treated constant ( $\neq f(V_{DS})$ )  
However  $\lambda$  has two components which shows that each is function of  $V_{DS}$

$$\lambda = \lambda_c + \lambda_m$$

$\lambda_c \rightarrow$  Related to channel length modulation

and  $\lambda_m \rightarrow$  Related to Field Dependent Mobility

We observe that  $x_c$  is function of depletion layer width,  $x_{dl}$ .

$$\text{As } x_{dl} = \sqrt{\frac{2k_s \epsilon_0}{q N_a} (\psi_s + V_{SB}')$$

$$\text{where } V_{SB}' = V_{SB} + V_{DS}$$

$$\text{or } x_{dl} = \sqrt{\frac{2k_s \epsilon_0}{q N_a} (\psi_s + V_{SB} + V_{DS})}^{1/2}$$

$$x_{dl} = K_1' (\psi_s + V_{SB} + V_{DS})^{+1/2}$$

$$\therefore \frac{dx_{dl}}{dV_{DS}} = K_1' \frac{d}{dV_{DS}} (\psi_s + V_{SB} + V_{DS})^{1/2}$$



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$$\begin{aligned} \text{or } \frac{dx_{de}}{dV_{DS}} &= K_1' \cdot \frac{1}{2} (\psi_s + V_{SB} + V_{DS})^{-1/2} \\ &= \sqrt{\frac{K_S \epsilon_0}{2q N_a} \frac{1}{(\psi_s + V_{SB} + V_{DS})}} \\ &= K_1'' (\psi_s + V_{SB} + V_{DS})^{-1/2} \\ &= \frac{K_1''}{(\psi_s + V_{SB})^{1/2}} \left[ 1 + \frac{V_{DS}}{\psi_s + V_{SB}} \right]^{-1/2} \end{aligned}$$

We know after Pinch off, increase of  $V_{DS}$ , increases  $x_{de}$

$$\therefore L_{\text{eff}} = L_{\text{Mask}} - x_{de}$$

$$\Delta \lambda_c = \left[ \frac{L}{L_{\text{eff}}} \cdot \frac{dx_{de}}{dV_{DS}} \right]$$

We have  $I_{DS}$  in Saturation

$$I_{DS} = \frac{1}{2} \beta' \left( \frac{W}{L_{eff}} \right) (V_{GS} - V_T)^2$$
$$= \frac{1}{2} \beta' \left( \frac{W}{L} \right) \frac{L}{L_{eff}} (V_{OV})^2$$

$$L_{eff} = L - x_{de} \quad \text{or} \quad \frac{L}{L_{eff}} = 1 + \frac{x_{de}}{L_{eff}}$$

$$I_{DS} = \frac{1}{2} \beta \left( 1 + \frac{x_{de}}{L_{eff}} \right) V_{OV}^2$$

We define  $\lambda_c = \frac{1}{L_{eff}} \cdot \frac{dx_{de}}{dV_{DS}}$

$$\text{or} \quad \lambda_c = \frac{1}{L - x_{de}} \cdot \frac{dx_{de}}{dV_{DS}}$$



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$$\text{or } \lambda_c = \frac{1}{L \left(1 - \frac{x_{dl}}{L}\right)} \cdot \frac{K_1''}{(\psi_s + V_{SB})^{1/2}} \left(1 + \frac{V_{DS}}{\psi_s + V_{DS}}\right)^{-1/2}$$

$$= \frac{1}{L \left[1 - \frac{1}{L} \sqrt{\frac{2k_s \epsilon_0}{qN_a} (\psi_s + V_{SB})}\right]} \times$$

$$\times \frac{K_1''}{(\psi_s + V_{SB})^{1/2}} \left(1 + \frac{V_{DS}}{\psi_s + V_{DS}}\right)^{-1/2}$$

$$\lambda_c = \frac{\lambda'}{L} = \frac{1}{L} \left[ \frac{1}{1 - \frac{1}{L} \sqrt{\frac{2k_s \epsilon_0}{qN_a} (\psi_s + V_{SB})}} \cdot \frac{K_1''}{(\psi_s + V_{SB})^{1/2}} \left(1 + \frac{V_{DS}}{\psi_s + V_{DS}}\right)^{-1/2} \right]$$



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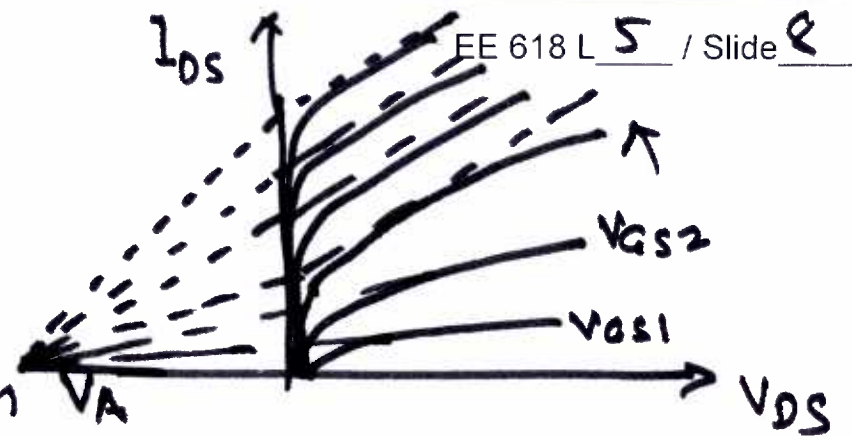
Mobility Dependent  $\lambda$  is given by

$$\lambda_m = \frac{\mu(0) - \mu(V_{DS})}{\mu(0) \cdot V_{DD}}$$

$\mu(0)$  = Low Field Mobility

$\mu(V_{DS})$  = Mobility at  $V(y) = V_{DS}$

However overall  $\lambda = \lambda_c + \lambda_m$



$$I_{DS} = \frac{1}{2} \beta' \left( \frac{W}{L} \right) (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$= I'_{DS} (1 + \lambda V_{DS}) = I'_{DS} \left( 1 + \frac{V_{DS}}{V_A} \right)$$

where  $V_A$  is called Early Voltage  $= \frac{1}{\lambda}$



Revisiting  $V_T$  relationship with  $V_{SB}$

We have

$$V_T = V_{T0} + \gamma \left[ (V_{SB} + 2\phi_F)^{1/2} - (2\phi_F)^{1/2} \right]$$

$$\begin{aligned} \gamma \frac{\partial V_T}{\partial V_{SB}} &= 0 + \gamma \left[ \frac{1}{2} (V_{SB} + 2\phi_F)^{-1/2} \right] \\ &= \frac{1}{2} \gamma (V_{SB} + 2\phi_F)^{-1/2} \end{aligned}$$

lets see variation of  $I_{DS}$  with  $V_{BS}$

We have

$$\frac{\partial I_{DS}}{\partial V_{SB}} = - \frac{\partial I_{DS}}{\partial V_{BS}}$$



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$$I_{Ds} = \frac{1}{2} \beta (V_{Gs} - V_T)^2 (1 + \lambda V_{Ds})$$

$$\cong \frac{1}{2} \beta (V_{Gs} - V_T)^2 \quad \text{Taking } \lambda \text{ Small}$$

$$\gamma \frac{\partial I_{Ds}}{\partial V_{SB}} = 2 \beta (V_{Gs} - V_T) \left( - \frac{\partial V_T}{\partial V_{SB}} \right)$$

$$= 2 \beta (V_{Gs} - V_T) \cdot \frac{1}{2} \gamma (V_{SB} + 2\phi_F)^{-1/2}$$

$$= 2 \beta (V_{Gs} - V_T) \frac{\gamma}{2 (V_{SB} + 2\phi_F)^{1/2}}$$

$$= \underline{2 \beta (V_{Gs} - V_T)} \cdot \eta$$

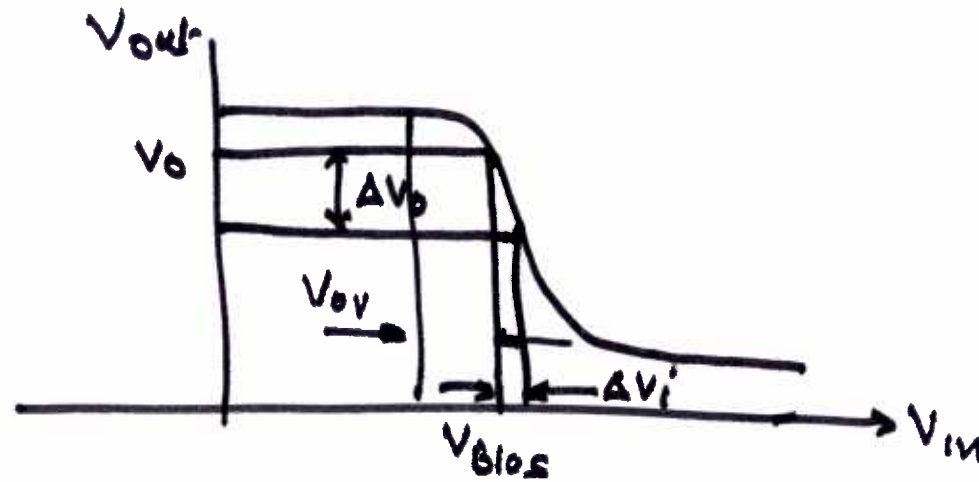
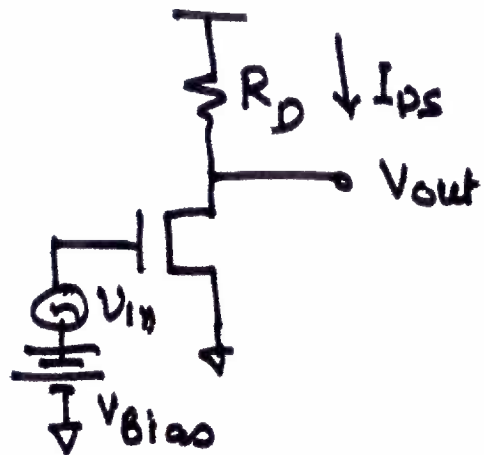
$$g_{mb} = g_m \cdot \eta$$

$$\text{where } \eta = \frac{\gamma}{2 (V_{SB} + 2\phi_F)^{1/2}} \cong 0.6$$



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Take a case of Simple MOS Amplifier



$V_{in} = V_{bias} + V_{signal}$   
 or  $\Delta V_{in} = V_{in} - V_{bias}$   
 $I_{Ds} = I_{Ds} + \Delta I_{Ds}$   
 $\Delta I_{Ds}$  is ac current

Clearly  $\Delta V_o =$  ac output  
 $= -\Delta I_{Ds} \cdot R_D$

But  $\frac{\Delta I_{Ds}}{\Delta V_{in}} = g_m$

$\therefore \Delta V_o = -g_m R_D \Delta V_{in}$   
 or Gain  $A_v = \frac{\Delta V_o}{\Delta V_{in}} = -g_m R_D$



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Representation of our Amplifier is :



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Since MOS Transistor in Saturation acts like  $V_{CCs}$

We write

$$\Delta V_{out} = \left[ \frac{1}{2} \beta V_{ov}^2 - \frac{1}{2} \beta (V_{ov} + \Delta V_{in})^2 \right] R$$

$$\text{or } \Delta V_{out} = -\frac{1}{2} \beta R \left[ V_{ov}^2 + \Delta V_{in}^2 + 2 V_{ov} \cdot \Delta V_{in} - V_{ov}^2 \right]$$

$$= -\frac{1}{2} \beta R \left[ \Delta V_{in}^2 + 2 V_{ov} \Delta V_{in} \right]$$

$$= -\frac{1}{2} \beta R 2 V_{ov} \cdot \Delta V_{in} \left[ 1 + \frac{\Delta V_{in}}{2 V_{ov}} \right]$$



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