Frequency Response of Amplifiers

Concepts of Poles & Zeros.

A Transfer F^n $A_v(s) = A_{vo} \cdot \dfrac{\left(\frac{s}{\omega_z} + 1\right)}{\left(\frac{s}{\omega_1}+1\right)\left(\frac{s}{\omega_2}+1\right)}$

ω_z is called 'zero' frequency at which $A_v(s) \to 0$

ω_1, ω_2 - - are Poles (Pole frequencies) at which $A_v(s) \to \infty$

In a typical MOS Amplifier we have around Two 'Poles' and One 'Zero'

Two Poles occurs from Input Side & output side

These can be termed as ω_{in} and ω_{out}

And ω_z is a 'zero' frequency occuring due to Feedback.

Ref. : Sedra, Smith & Chandorkar
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SINGLE POLE TRANSFER FUNCTION

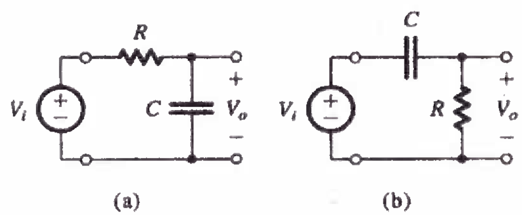

FIGURE 1.22 Two examples of STC networks: (a) a low-pass network and (b) a high-pass network.

TABLE 1.2 Frequency Response of STC Networks

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\frac{K}{1+(s/\omega_0)}$	$\frac{Ks}{s+\omega_0}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\frac{K}{1+j(\omega/\omega_0)}$	$\frac{K}{1-j(\omega_0/\omega)}$
Magnitude Response $\lvert T(j\omega)\rvert$	$\frac{\lvert K\rvert}{\sqrt{1+(\omega/\omega_0)^2}}$	$\frac{\lvert K\rvert}{\sqrt{1+(\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega=0$ (dc)	K	0
Transmission at $\omega=\infty$	0	K
3-dB Frequency	$\omega_0 = 1/\tau$; $\tau \equiv$ time constant; $\tau = CR$ or L/R	
Bode Plots	in Fig. 1.23	in Fig. 1.24

BODE PLOTS of STC

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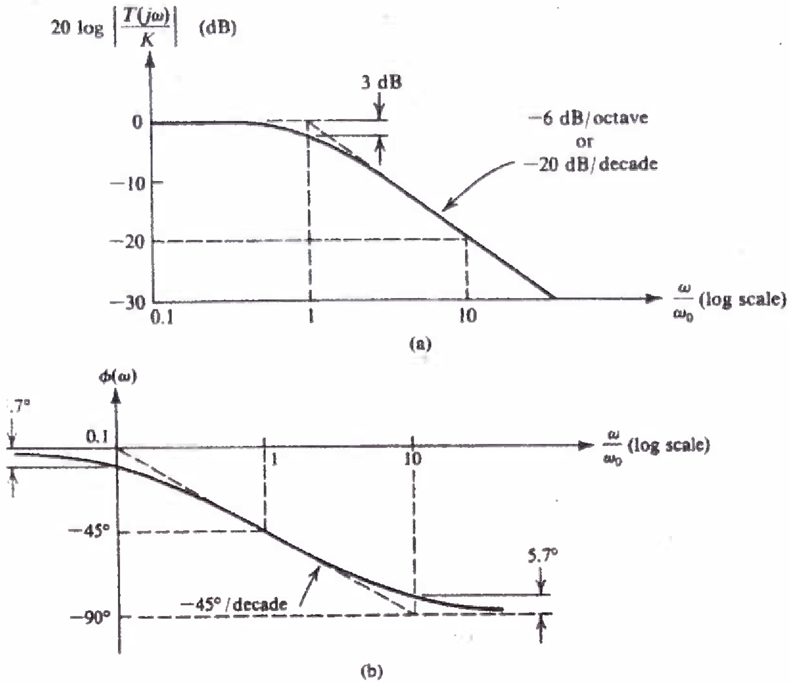


FIGURE 1.23 (a) Magnitude and (b) phase response of STC networks of the low-pass type.

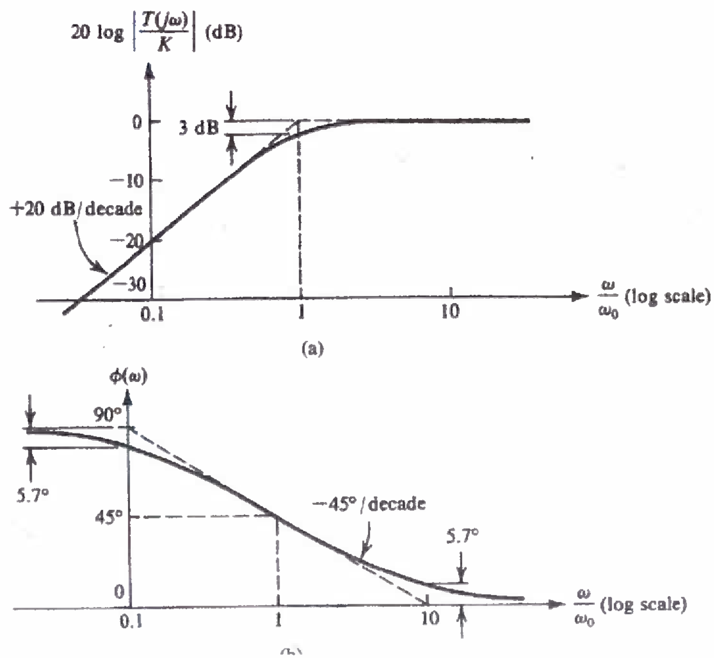
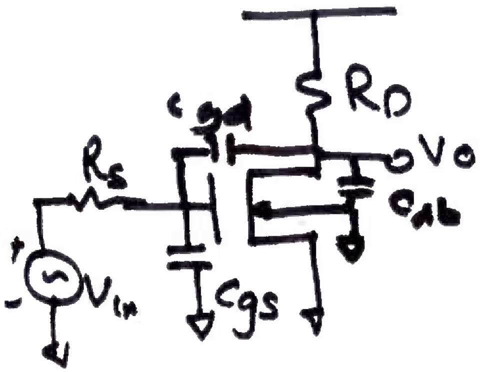
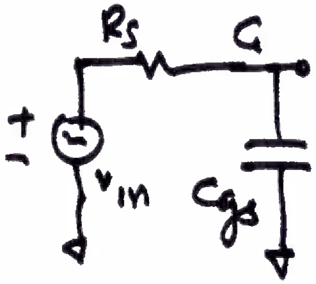


FIGURE 1.24 (a) Magnitude and (b) phase response of STC networks of the high-pass type.

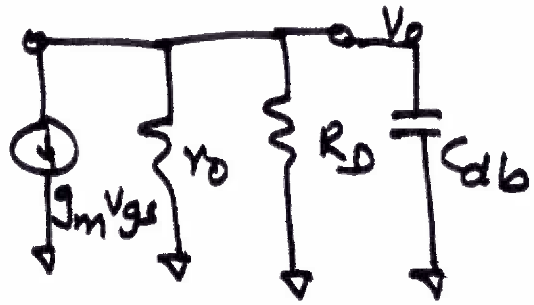
Input pole & Output Pole



$$\omega_{in} = \frac{1}{R_s C_{gs}}$$



input



output

$r_o \| R_D \cong R_D$

& $$\omega_{out} = \frac{1}{R_D C_{db}}$$

However in equivalent ckt of Amplifier we also have capacitance C_{gd}

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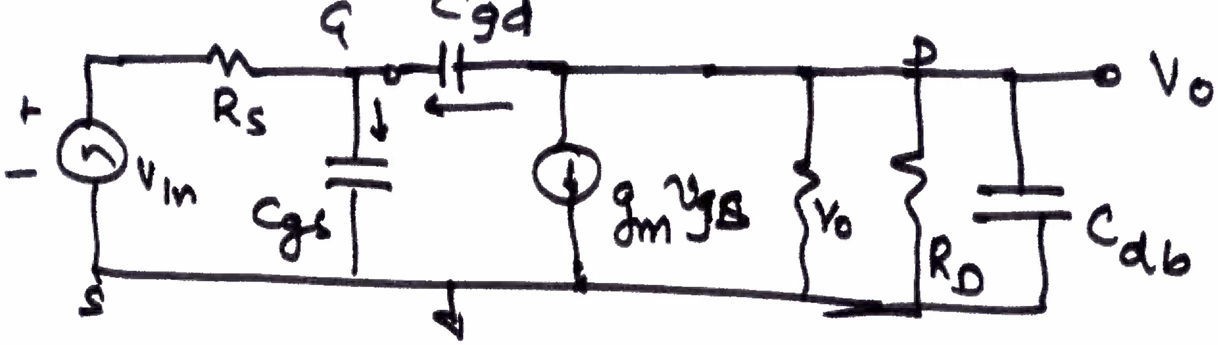
Then Eq. Ckt looks like



Assume
$r_o \| R_D = R_D$
Also C_{db} effect is neglected

$$v_{gs} = \frac{\frac{1}{C_{gs} \cdot s}}{\frac{1}{sC_{gs}} + R_s} v_{in} = \frac{v_{in}}{1 + s \cdot R_s C_{gs}}$$

C_{gs} may include C_{gb} as it may occure in Saturation.

Then $$A_v'(s) = \frac{v_o(s)}{v_{gs}(s)} = -\{g_m R_D\}\left\{\frac{1 - \frac{C_{gd} \cdot s}{g_m}}{1 + s \cdot C_{gd} \cdot R_D}\right\}$$

$$A_v(s) = \frac{V_o(s)}{V_{in}(s)} = -(g_m R_D) \frac{(1 - s \cdot C_{gd}/g_m)}{(1 + s R_s C_{gs})(1 + s \cdot R_D C_{gd})}$$

Approximately looking at denomenator we see

$$D = g_m (1 + s R_s C_{gs})(1 + s R_D C_{gd})$$

$$\cong (1 + s R_s C_{gs})(g_m + s \cdot g_m R_D C_{gd})$$

$$= (1 + s R_s C_{gs})(g_m + (-A_{vo}) s \cdot C_{gd})$$

$$\cong 1 + s \{ C_{gs} + (1 + A_{vo}) C_{gd} \} R_s$$

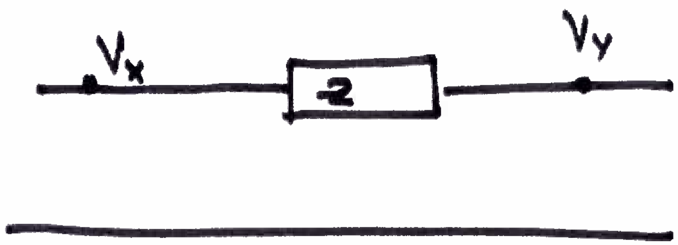
$$= 1 + R_s (C_{gs} + (1 + g_m R_D) C_{gd}) \cdot s$$

$$= 1 + R_s C_{in} \cdot s \qquad C_{in} = C_{gs} + (1 + g_m R_D) C_{gd}$$

$$\therefore \omega_{in} = \frac{1}{R_s C_{in}}$$

Similar Result we can attain by using Miller's Theorem:

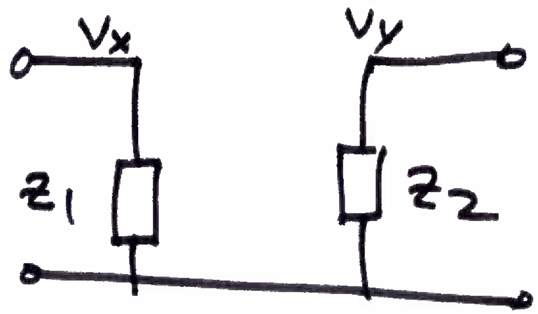
Given



where

$$A = \frac{V_y}{V_x} \quad (\text{Gain } F^n)$$

we can convert this to



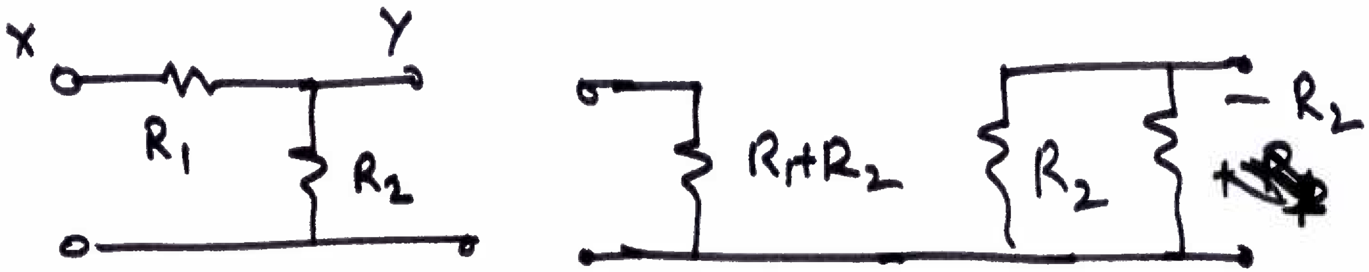
where

$$Z_1 = \frac{Z}{1-A}$$

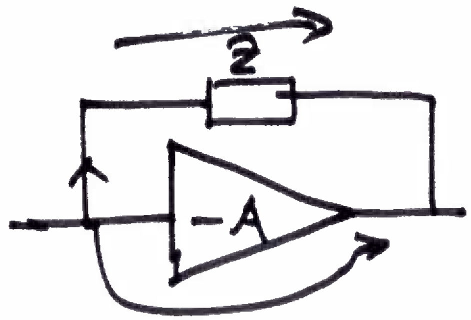
& $$Z_2 = \frac{A}{1-A} \cdot Z$$

If Z is capacitive C_{xy}

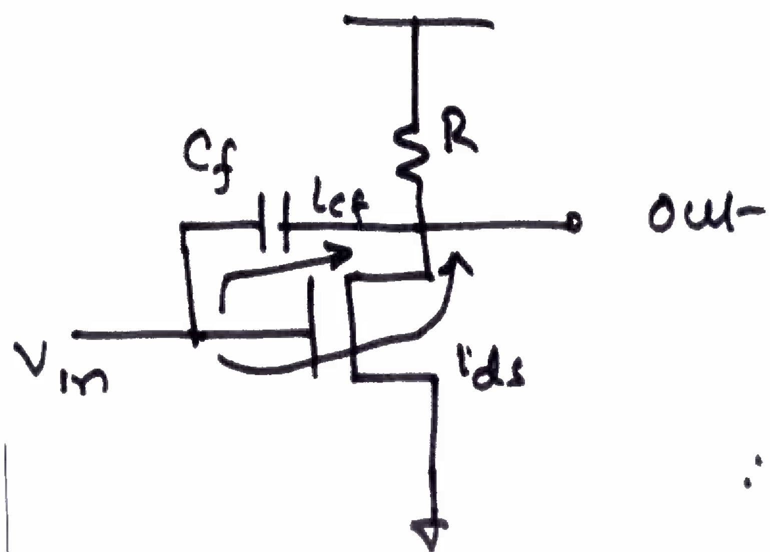
Limitations of Miller's Theorem



We must have Two Paths from Input to Output for validity of Miller's theorem.



Continuing similar argument, we can see that a 'zero' of Transfer Function is only possible if we have two paths from Input to Output and at a frequency the value of current in two paths are equal in magnitude and 180° out of phase.

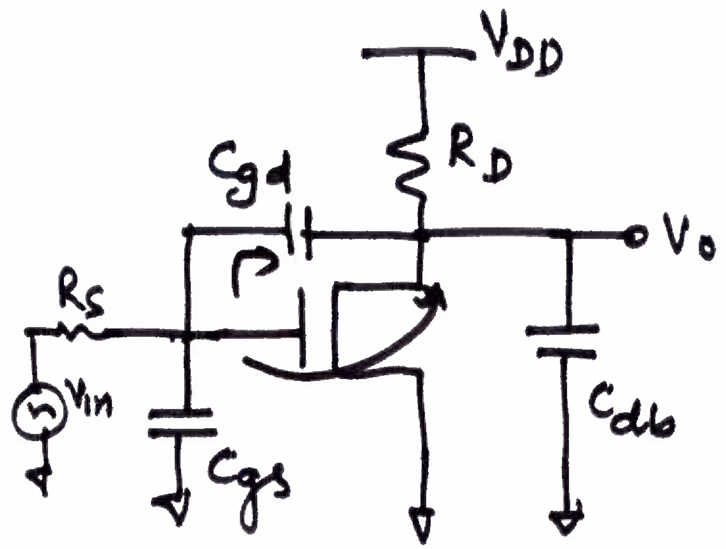


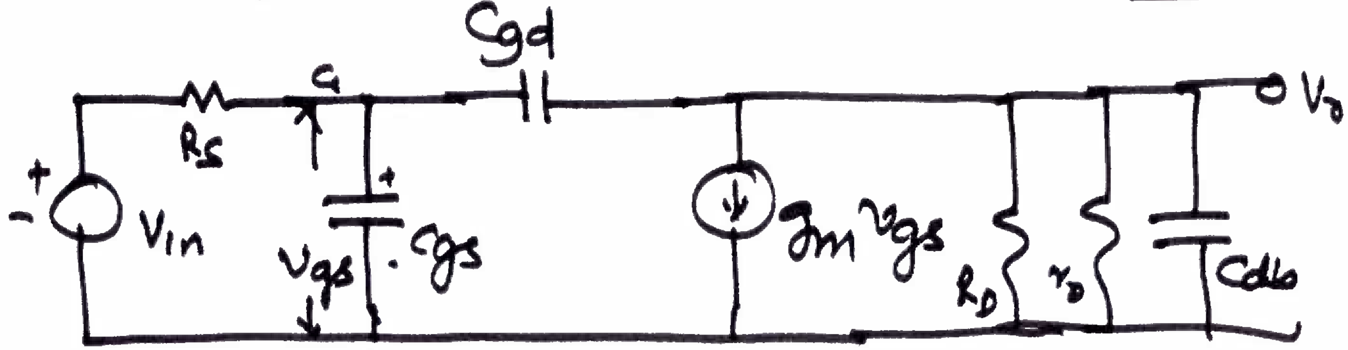
This is due to effect of "Feed Forward"

$$\boxed{i_{cf} = -i_{ds}}$$

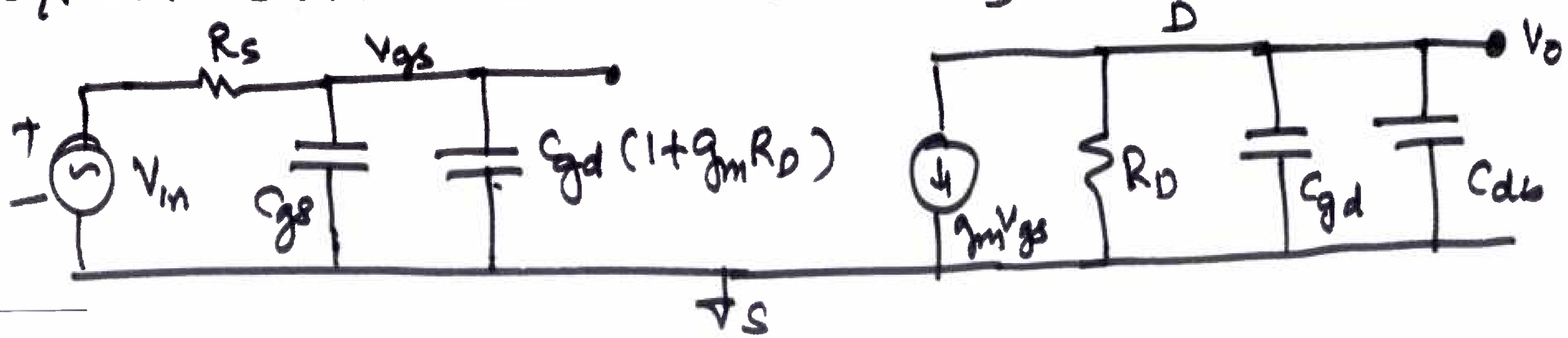
$\therefore$ $i = 0$ $\therefore$ $V_{out} = 0 \rightarrow$ 'zero' Occurance

Returning to our problem of Frequency response of a CS Amplifier, whose HF model can be shown as with eq ckt as —





Using DC Gain expression $A_{Vo} = -g_m R_D$, we see that Eq. Ckt could be shown using Miller's theorem as :—



Then

$$\omega_{in} = \frac{1}{R_s\left[C_{gs} + (1+g_m R_D)C_{gd}\right]}$$

& $$\omega_{out} = \frac{1}{R_D(C_{gd} + C_{db})}$$

And Transfer F^n can be written as

$$A_V(s) = \frac{A_{Vo}}{\left(1+\frac{s}{\omega_{in}}\right)\left(1+\frac{s}{\omega_{out}}\right)}, \quad A_{Vo} = -g_m R_D$$