Self-Phase Modulation

\[ L_D \gg L, \quad L \gg L_{NL} \]

NLS: \[
\frac{\partial U}{\partial z} = -j \frac{e^{-\alpha z}}{L_{NL}} |U|^2 U
\]

Let \[ U = V e^{j\phi_{NL}} \]

\[
\frac{\partial U}{\partial z} = \frac{\partial V}{\partial z} e^{j\phi_{NL}} + jV e^{j\phi_{NL}} \frac{\partial \phi_{NL}}{\partial z}
\]

\[
= -j \frac{e^{-\alpha z}}{L_{NL}} |V|^2 V e^{j\phi_{NL}}
\]

\[ L_{NL} = \frac{1}{\gamma P_0} \]
\begin{align*}
\frac{\partial v}{\partial z} &= 0, \quad \frac{\partial \phi_{NL}}{\partial z} = -\frac{e^{-\alpha z}}{L_{NL}} |v|^2 \\
\left\{ \right. \\
\text{Amplitude does not change} \\
\text{Phi}_NL \text{ changes with distance} \\
\frac{v}{U(z, T)} = U(0, T) \\
\phi_{NL} &= \int_0^L -\frac{e^{-\alpha z}}{L_{NL}} |v|^2 \, dz \\
&= -\left\{ \frac{1}{L_{NL}} \right\} \left\{ \frac{1 - e^{-\alpha z}}{\alpha} \right\} \\
&= \left\{ \frac{\alpha}{L_{NL}} \right\} \left\{ \frac{1 - e^{-\alpha z}}{\alpha} \right\} \left\{ \frac{1}{L_{NL}} \right\} \\
&= \text{Left expression}
\end{align*}
Spectral Broadening due to Non-linearity

Pulse Spectra

Distance
Signal and spectrum

Dispersive Regime

Distance

Pulse shape

Spectrum

Non-linear Regime

Distance

Time Pulse

Spectrum
\[ \beta_2 > 0 \quad \text{Normal dispersion} \quad \text{for } \lambda < 1300 \text{ nm} \]

\[ \beta_2 < 0 \quad \lambda > 1300 \text{ nm} \]

Anomalous dispersion

\[ z_2 > z_1 \quad \beta_2 > 0 \]

\[ z_1 > 0 \]

\[ z = 0 \]

\[ z = 1550 \text{ nm} \]

\[ z_2 > z_1 \quad \beta_2 < 0 \]

Anomalous dispersion
Dispersion

Soliton

Undistorted
Define $u = N U \rightarrow U = u / N$

Anomalous dispersion $\beta_2 < 0$, $\text{sgn}(\beta_2) = -1$

\[
\frac{\partial u}{\partial \xi} + j \frac{1}{2} \frac{\partial^2 u}{\partial \xi^2} + j \frac{1}{4} u^2 u = 0
\]

Assuming $u(\xi, \tau) = V(\tau) e^{i \phi(\xi, \tau)}$

\[
\phi(\xi, \tau) = -K \xi + \delta \tau
\]

- phase const
- $\delta = 0$
- frequency shift

$\phi(\xi, \tau) = -K \xi$
\[
\frac{d^2 V}{d \tau^2} = 2V(K - V^2) \times 2 \frac{dV}{d\tau}
\]

\[
\int 2 \frac{dV}{d\tau} \frac{d^2 V}{d\tau^2} d\tau = \int 2V(K - V^2) 2 \frac{dV}{d\tau} d\tau
\]