FIBER OPTICS

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Lecture: 23

Optical Receivers-
Performance Analysis and the EYE-diagram
When optical signal from the output of an optical fiber is incident onto a photo-detector material, the material absorbs the power in the optical signal to generate electron-hole pairs in the material which, in turn, give photo-current. So, if there is a random reception of unbiased digital data in the optical form, the quantity of interest is the average power in the optical signal because any power meter is capable of measuring only the average power in an input signal. Therefore, there has to be some minimum average power required in the signal for its reliable detection in the optical domain.

Let us assume, ‘$P_1$’ be the average incident power onto the photo-detector when logic ‘1’ is received and ‘$P_0$’ be the average power incident onto the detector when logic ‘0’ is received. Also, let ‘$I_1$’ and ‘$I_0$’ be the generated photo-currents corresponding to the two levels ‘1’ and ‘0’ respectively. Without compromising any generality, let us assume that $P_0=0$ such that $I_0=0$. That means, there is no incident power on the photo-detector for logic ‘0’. If ‘$R$’ be the responsivity of the photo-detector then:

$$I_1 = R \cdot P_1$$

(23.1)

$$I_0 = R \cdot P_0 = 0$$

(23.2)

The average power received is given by the following relation:

$$\overline{P}_{\text{received}} = \frac{P_1 + P_0}{2} = \frac{P_1}{2}$$

$$\Rightarrow P_1 = 2\overline{P}_{\text{received}}$$

(23.3)

The measure of the noise in the reception of the two logic levels ‘0’ and ‘1’ can be ascertained from the variances in the generated photo-current in the two levels. For the logic ‘0’ reception, there is no incident power and the thermal noise component dominates the noise component in the output signal of the photo-detector. So, the total variance in the ‘0’ level (denoted by ‘$\sigma_0^2$’) is given by:

$$\sigma_0^2 = \sigma_T^2$$

(23.4)

‘$\sigma_0^2$’ is the variance that characterises the thermal noise component in the reception. For the logic ‘1’ level, the noise component in the output signal from the photo-detector is composed of two types of noise- shot noise and thermal noise. Therefore, the total noise variance (‘$\sigma_1^2$’) is the sum of the variances of shot noise (‘$\sigma_s^2$’) and thermal noise (‘$\sigma_T^2$’). That is:

$$\sigma_1^2 = \sigma_s^2 + \sigma_T^2$$

(23.5)

If the average power of the received signal is large, the thermal noise becomes almost negligible in comparison to the shot noise. Using equations 20.5, 20.8 and 23.1 for a normal p-i-n photo-detector with responsivity ‘$R$’ and a load resistance ‘$R_L$’, we may write:
The Q-parameter for the above photo-detector under discussion can be expressed as:

\[ Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} = \frac{I_1}{\sigma_1 + \sigma_0} = \frac{2R\overline{P}_{\text{received}}}{(\sigma_2^2 + \sigma_T^2)^{1/2} + \sigma_T} \] (23.8)

By rearranging equation 23.8 and substituting the values of the noises, we obtain:

\[ \overline{P}_{\text{received}} = \frac{Q}{R}(qBQ + \sigma_T) \] (23.9)

The above relation helps us to calculate the minimum average power required to achieve the desired bit-error-ratio (BER). For the maximum acceptable BER (which is about 10^{-9}), the value of Q must be equal to or greater than 6.

For the thermal noise dominated operation of the photo-detector, \( qBQ \ll \sigma_T \) and so:

\[ \overline{P}_{\text{received}} \approx \frac{Q}{R} \sigma_T = \left[ \frac{Q}{R} \frac{4KT}{R_t} \right]^{1/2} \sqrt{B} \] (23.10)

From the above equation, we see that:

\[ \overline{P}_{\text{received}} \propto \sqrt{B} \] (23.11)

The above relation shows that at low optical power level operation, the average power required to achieve the desired BER is directly proportional to the square root of the bandwidth of operation. Since bandwidth is directly proportional to the data rate of transmission, we may say that as the data rate of information transmission increases, the minimum power required to achieve the desired BER also increases. However, this increase is not rapid because the power varies as square root of the bandwidth. This relation holds true, also when, the transmitter and receiver stations are very far from each other (as already discussed earlier) thereby making even the high power operation domain a low-power one, due to attenuation of signal power.

When the transmitter and receiver are near to each other, the operation becomes a shot noise dominated operation. For a shot noise dominated operation \( qBQ \gg \sigma_T \), and equation 23.9 modifies to:

\[ \overline{P}_{\text{received}} \approx \frac{qBQ^2}{R} \] (23.12)
From the above relation we see that:

$$\overline{P}_{\text{received}} \propto B \quad (23.13)$$

This relation shows that as the domain of operation transits from the thermal noise dominated to the shot noise dominated domain, the minimum required power to achieve the desired BER becomes directly proportional to the linear power of the bandwidth. This means, as the data rate of information transmission increases, the minimum power required to achieve the desired BER also increases in the same proportion.

If we design the best possible optical receiver which introduces no external noises (thermal and dark-current noises) into the detected signal such that whatever the noise present is only due to the statistical variations of the photons, the minimum number of photons required to be detected per bit for achieving the desired BER is called the **quantum limit of detection** of the receiver. Since the thermal noise is absent, there would be no output for a logic ‘0’ transmission in which there is no input incident power. This means logic ‘0’ level would be detected very accurately. However, when optical power is transmitted for a logic ‘1’ transmission and if even a single photon is detected, the output at the receiver can be assigned as a ‘1’ being transmitted because of the absence of thermal noise. But, if due to the statistical nature of the transmitted photons even that single photon fails to get detected within a bit-duration, the output of the receiver would be assigned ‘0’ and consequently it would lead to a bit error. So, the quantum limit of detection would be the minimum number of photons which are required to be transmitted to generate at least one detected photon.

For calculating the quantum limit of detection, let us assume $P(t)$ be the transmitted optical power during a bit-interval and $\eta$ be the efficiency factor related to the generation of electron-hole pairs in the material of the photo-detector. If $h\nu$ be the energy of one photon, the number of electron-hole pairs generated ($N$) is given by:

$$N = \frac{\eta}{h\nu} \int_0^\tau P(t)dt \quad (23.14)$$

Here we assume $\tau$ be the duration of a bit and the transmitted power be varying with time during the bit interval and this justifies the use of the integral sign. The above equation, however, gives us the average number of electron-hole pairs generated during a bit interval because the generation of electron-hole pairs by absorption of photons is a probabilistic process which has a Poisson’s distribution. Therefore, the probability of ‘$n$’ electron-hole pairs being generated during any bit interval is given by the Poisson’s distribution as shown below:

$$P(n) = N^n \frac{e^{-N}}{n!} \quad (23.15)$$
However, inspite of optical power transmission, if no electron-hole pairs were generated during the bit interval then there would be no output and the bit would be assigned ‘0’ at the receiver. This would then cause a bit error. Therefore, the probability that no electron-hole pairs are generated during a bit interval is given by substituting $n=0$ in the equation 23.15. That is:

$$P(0) = \frac{N^0 e^{-N}}{0!} = e^{-N} \quad (23.16)$$

Equation 23.16, in fact, gives the value of BER for the receiver under study. For an acceptable system, the BER should at most be $10^{-9}$. That is:

$$e^{-N} \leq 10^{-9} \Rightarrow N \approx 21 \quad (23.17)$$

So for the best possible receiver with no thermal noise, there should be atleast 21 photons, on average, incident onto the photo-detector during one bit interval for an acceptable BER. This is the quantum detection limit of the receiver. However, in practice, this is too small a number and the actual number of photons per bit ranges from a few hundred to a few thousand. Also, in practice, the quantum limit of detection of a practical receiver would be a lot smaller than 21 because of the presence of thermal and other kinds of noises which deteriorate the receiver performance.

The quantum limit of detection gives the average number of photons that required to be transmitted per bit-duration for achieving the desired BER. Therefore, if the bit-duration decreases (i.e. if data rate increases), the quantum limit of detection (number of photons per bit) would remain the same but the number of photons per second would increase and the average power would, thus, increase. Thus the average power is proportional to the data rate which satisfies the relationship given in 23.13. Quantum limit of detection is one of the important characteristics of evaluation of an optical receiver. Another such important characteristic is the inter-symbol interference (ISI) in an optical receiver.

Due to the finite bandwidth of the optical receiver, the sharp edges of transitions from logic ‘0’ to logic ‘1’ or vice-versa, do not appear sharp enough at the output of the receiver. The optical receiver has a finite rise time and fall time which causes the sharp transitions to blur out. Coupled with this, is the finite settling time required for the transition voltage to settle to the steady value of the logic level voltage. If the subsequent bit arrives within this settling time, there is said to be interference between this logic bit and the preceding bit whose transition voltage has not yet settled to a steady state value. The voltage value of this subsequent value would hence be the superposition of the two transition voltages. This phenomenon is called inter-symbol interference in the transmission. Therefore, when unbiased data is transmitted, the detected output signal at the receiver is shown in the figure below:
In the process of detection, we do not integrate the optical power received within an optical pulse. Instead, the pulses are converted to equivalent photo-current and then to the corresponding voltages and at particular instants of time \((n_1, n_2, \ldots)\), at which a bit is expected, the receiver checks (samples) the amplitude of the signal. If the amplitude at the sampling instant is greater than the threshold value, the bit received is declared as logic ‘1’ and if the amplitude is lower than the threshold value, the bit is declared as logic ‘0’. However, the important point to note here is that, due to the presence of ISI, the received signal which, ideally, would have looked like the rectangular waveform in the above figure, actually looks like the waveform shown by ‘with ISI’ in the above figure. It is evident that the presence of ISI deteriorates the shape of the actual pulse and even the amplitude of the pulse. At instant ‘\(n_6\)’, even before the signal could have reached the steady state value of the bit ‘0’ (as in instants \(n_2, n_3\)), the next bit ‘1’ arrives and the signal starts to rise. This problem persists for the instants \(n_7, n_8, n_9\) etc. wherever there is a complementary bit between two similar bits.

It is, hence, highly desirable that an appropriate measurement strategy be developed to measure the amount of signal distortion created due to ISI. In order to do that, we shall investigate all possible bit transitions in the out data signal. These transitions are shown below in the figure 23.2, which shows all possible combinations of 3-bit binary data and the corresponding detected signal at receiver.
Figure 23.2: Different 3-bit input signal bit-patterns and corresponding detected signals

The 'system' shown in the above figure is, in fact, the receiver block to which the optical input signal is fed. Therefore, depending on the bit-patterns in the input data stream, the detected signal would be combinations of the detected signals shown in the above figure.

The aim of this analysis is to devise a measurement strategy to measure the amount of distortion that occurs in the detected signal at the receiver output due to the finite bandwidth of the receiver and also due to the presence of noise. In the above figure, we have not shown the actual noisy receiver output. So, just to be familiar with the actual output received signal waveform, an arbitrary bit-pattern '01001' is shown in the figure below.

Figure 23.3: Actual noisy output waveform of receiver

The noise in the signal is, in fact, a function of the amplitude of the signal in case of the shot-noise dominated operation domain. So, to measure the distortion in the received signal a special technique called the 'eye-diagram technique' is used. The experimental set-up required for the measurement of distortion in the received signal by this technique is shown in the figure 23.4 below. The PN-sequence generator generates an unbiased stream of bits which serves as an input to the...
receiver (system under study) and the output of the receiver is then viewed on an oscilloscope. The clock of the oscilloscope is externally triggered so that the clocks of the PN-sequence generator and the oscilloscope are synchronized with each other.

[Image: Diagram of experimental setup]

**Figure 23.4**: Experimental set-up for measurements on signal-distortion

The synchronizing of the two clocks causes the output waveform of the oscilloscope to be a superposition of all possible waveforms shown in figure 23.2 and the resultant pattern looks like:

[Image: Eye-diagram of receiver]

**Figure 23.5**: Eye-Diagram of the receiver

The above pattern generated as a result of superposition of different possible bit patterns looks like the eye and is, hence, called as the eye diagram. The broadening of the lines is due to the fluctuations in optical intensity due to the noise that is present in the output signal from the system. Without the noise, the pattern would have looked like the one shown in the figure 23.6 below. The solid lines show the logic ‘0’ and ‘1’ levels and indicate the mean level of the bit current (or voltage).
The eye-diagram shown in figure 23.5 above, indeed, gives a visual sense of the quality of the received data signal. The eye-diagram facilitates the calculation of a few important parameters pertaining to the performance of the system. The first such parameter is the Q-parameter of the receiver which, in turn, determines the BER of the data reception. From equation 22.14, we see that the value of Q can be calculated from the mean values of photo-current of the two logic levels '0' and '1' which are $I_0$ and $I_1$ respectively, as shown in figure 23.6. These two values can be easily determined, by observations, from the eye-diagram as shown below:

If the noise in the two levels is assumed to be Gaussian in nature, the peak-to-peak deviation is about 5 times the variance of the distribution. So, by measuring the peak-to-peak deviations in the two levels (as shown in the figure 23.7), the variance of the signal amplitude in the two levels can be calculated using the following relations.

\[ d_1 = 5\sigma_1, \quad d_0 = 5\sigma_0 \]
Once, the mean levels of current and the variances are determined, the Q-parameter can be easily determined using equation 22.14 which is shown below:

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$$

This value of Q can then be plotted on the BER Vs Q curve of the optical receiver and the corresponding value of BER can be determined graphically.

Let us now look into another important aspect that the eye-diagram depicts. As the SNR of the receiver decreases, the two levels ‘I_0’ and ‘I_1’ come closer to each other in the vertical direction. The decrease in SNR is, in fact, a decrease in the value of Q which, in turn, indicates an increase in the BER. Said reversely, any reduction in the relative amplitude of the two logic levels, in fact, indicates a reduction in the SNR of the receiver.

Since the clock signals of the PN-sequence generator and the oscilloscope are synchronized with each other, the time interval between the points A and B (in figure 23.7) is equal to the duration of either 1 bit or 3 bits. If the received signal (on Y channel) is perfectly synchronized with the clock, the two points A and B are perfectly stable points. In case of practical systems the clock signal is, in fact, extracted out of the received signal. If this clock is perfectly stable, the two points A and B are absolutely steady as a function of time as shown in figure 23.6. But if there arises even a slight mismatch between frequencies of the clock and the received data signal, some kind of a jitter in the timing of the signal is produced and the points A and B would be deviated in the horizontal direction from their steady position. That is, the horizontal width of the region at point A and B is, in fact, a measure of the clock-jitter as shown in the figure 23.7. As the clock-jitter increases, the width of this region increases and the eye is said to close in horizontally. In other words, the horizontal closing of the eye indicates an increase in the amount of mismatch between the clock frequency and the frequency of the received signal.

From the above discussion, we can say that the eye-diagram reveals two very important parameters of the receiver: Q-parameter and the clock-jitter. Hence, the eye-diagram is, in fact, a very powerful and important tool to determine the quality of performance of the receiver. That is why, after an optical receiver is commissioned into the communication system, the eye-diagram test of the receiver is done to ensure optimal performance parameters and also measure the amount of discrepancies between the desired and the actual performance characteristics.