Lecture: 16
Optical Sources
Introduction to LASER
The three distinct phenomena that occur inside a semiconductor material are the absorption, spontaneous emission and the stimulated emission phenomena. Spontaneous emission occurs without the influence of any external stimulating agent and hence is named so. On the other hand, absorption and stimulated emission occur in the presence of external stimuli and, hence, are called as stimulated processes. This external stimulus is usually light, which is allowed to be incident on the material. However, even when the material is illuminated, spontaneous emissions may occur if there are electrons in the excited state.

So, when light is incident on a semiconductor material, three distinct phenomena may occur simultaneously - spontaneous emission, absorption and stimulated emission. But, as we have already discussed in our earlier sections, at thermal equilibrium, the rate of absorption must be equal to the total rate of emission. This notion led us to a mathematical expression for the resultant photonic flux density that emanates from the semiconductor material which is given by:

\[ \rho(v) = \frac{A_{21} B_{21}}{B_{12} N_1 N_2} \]  

(16.1)

The terms in the above equation have their usual meanings as already discussed in context of the two energy levels \( E_1 \) and \( E_2 \) (\( E_2 > E_1 \)). We also know that, 

\[ \frac{N_1}{N_2} = e^{\frac{hv}{kT}} \]  

(16.2)

Substituting this in equation 16.1, we can re-write equation 16.1 as:

\[ \rho(v) = \frac{A_{21} B_{21}}{B_{12} e^{\frac{hv}{kT}} - 1} \]  

(16.3)

The Rayleigh-Jeans’s law describes the spectral energy distribution in the radiation emitted by a ‘Black-Body’. This law is mathematically expressed as:

\[ \rho(v) = \frac{8\pi n^3 v^3}{c^3} \cdot \frac{1}{e^{\frac{hv}{kT}} - 1} \]  

(16.4)

Here, ‘\( n \)’ is the refractive index of the medium in which the radiation travels. The photon flux emanating from the material under study which is at thermal equilibrium at temperature ‘\( T \)’ may be considered similar to the radiations from a black body at thermal equilibrium at the same temperature. This leads us to compare equation 16.3 with 16.4 and, by doing so we may write the following expressions:

\[ \frac{A_{21}}{B_{21}} = \frac{8\pi n^3 v^3}{c^3} \]  

(16.5)
And,

$$B_{12} = B_{21} \quad (16.6)$$

Equation 16.6 suggests that there is an equal probability of occurrence of absorption and stimulated emission in a semiconductor material when light is incident on it. However, in nature we do not see the occurrence of stimulated emission but absorption is a more common phenomena observed when light is incident on a material. The reason for this lies in the Boltzmann’s distribution of equation 16.2. In practical situations, at thermal equilibrium, the electron density in the excited state is negligible. So the rate of stimulated emission is almost negligible in comparison to the rate of absorption. That is why, we see absorption phenomena occurring more often than an emission phenomena. Emission phenomenon can occur only if there is electron density in the excited state, which occurs only when the material is energized. Hence, although the absorption and the stimulated emission processes are equally probable, the absorption process dominates over the stimulated process at thermal equilibrium.

The above discussion initiates a basic thought in the mind that, what would be the outcome if, by some means, the population (density) of electrons in the excited state is made much more in comparison to that in the ground state? The answer would be: the stimulated emission process would dominate over the absorption process. So, for stimulated emission to occur, first, we have to create a considerable density of electrons in the excited state from the ground state so that the ground state density of electrons becomes negligible in comparison to that in the excited state. That is, we have to invert the thermal equilibrium distribution of electrons in the material. This process is called as population inversion. This would cause the emissive process to be dominant over the absorptive process and the net result would be occurrence of stimulated emission from the material when it is illuminated. However, the net radiation from the material is not only due to the stimulated emission but also due to the spontaneous emission which emits highly incoherent light.

If we roll back to our discussion on the stimulated phenomena occurring between two energy levels $E_1$ and $E_2$ ($E_2 > E_1$), we would realise that the photon that is absorbed or emitted out, also has an energy which is equal to the energy difference between the two levels i.e. $(E_2 - E_1)$. In other words, only those photons whose energy equals the above energy difference are either absorbed or given out. One can refer to this observation as some kind of a resonance phenomenon which involves the emission or absorption of photons with a particular frequency ‘$\nu$’ where this corresponds to the frequency of the above energy difference ($h\nu = E_2 - E_1$). The direction of energy flow, however, depends on the magnitude of energy in the incident photon and the electrons in the material. In practical situations, at thermal
equilibrium, we see energy flow from incident photons to the electrons in the material and in the stimulated emission we see the energy flow from the electrons to the emitted photons. These emitted photons owe their origin to the incident photons and so are completely coherent with the incident photons.

Whenever, light is incident on a material, three simultaneous processes occur in the material, viz. absorption, spontaneous emission and the stimulated emission. However, the contribution of the spontaneous emission over the stimulated emission in the net radiation from the material can be ascertained from a very crude quantitative calculation as shown below. We already know that the rate of stimulated emission is directly proportional to the product of the Einstein coefficient, $B_{21}$ and the incident photonic flux density, $\rho(\nu)$; whereas the spontaneous emission is proportional to the Einstein coefficient, $A_{21}$. Hence, to find the contribution of one over the other we have to find a ratio between these proportionalities. This ratio can be calculated from equation 16.3 above, and is given as:

$$\frac{B_{21}\rho(\nu)}{A_{21}} = \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$  \hspace{1cm} (16.7)

For a radiation having wavelength, $\lambda$=1µm, the exponential term in the denominator of equation 16.7 turns out to approximately $e^{50}$, at room temperature. Thus the ratio:

$$\frac{B_{21}\rho(\nu)}{A_{21}} \approx e^{-50}$$  \hspace{1cm} (16.8)

Since $e^{-50}<<1$, the equation 16.8 suggests the fact that stimulated emission is practically non-existent at room temperature, under thermal equilibrium. Hence the net radiation from the material is highly incoherent since it takes place due to spontaneous emissions which are incoherent and are called as black-body radiations.

As we already discussed, the stimulated emission process is a cumulative process which causes multiplication in the number of emitted photons. On the other hand, spontaneous emission process is indebted to the natural tendency of the excited electron to go down to the ground state by release of energy. Hence, the carrier life-time against spontaneous emission is much larger than the carrier life-time against stimulated emission which is a stimulated process. This fact causes the stimulated emission to dominate over the spontaneous process, as soon as photons are made incident onto the material and the emission predominantly is stimulated rather than spontaneous. Therefore, in the subsequent discussions about emission of photons, we shall emphasize only on the stimulated emission process.

We are already familiar with the fact that to have a stimulated emission of photons from a material, there has to be a population inversion of the electrons created apriori. To create population inversion, the electrons have to be supplied with the
required amount of energy to cause them to migrate to the excited state. The difference in energies between the ground state and the excited state corresponds to the energy of the photon that would be emitted out. In other words, the initial energy to be supplied to the electrons to migrate to the excited state thus equals the energy of the photon that is emitted out. In particular, the frequency of the incident photon (to cause absorption) has to be exactly equal to the energy of the emitted photon (by stimulated emission). That is, to have a coherent emission we must already have coherent photons incident on to the material. This is practically not possible. In other words, this fact suggests that a simple two energy level system of $E_1$ and $E_2$ ($E_2 > E_1$) would, rather, be inappropriate to realise a LASER action in practice. So, the obvious alternative that comes to the mind is to have an energy level system which has more than two energy levels. This type of a system is shown in figure 16.1 below.

![Figure 16.1: System of more than two energy levels](image)

As seen from the above figure, a beam of spatially incoherent photons is made to illuminate the material, which causes the electrons in the ground state to migrate to an excited state ($E_3$) that is higher in energy than the required excited state ($E_2$). This excited electron then quickly jumps to $E_2$ by a spontaneous process of release of energy and then is released out as a photon of the required nature by decaying to the ground state as a result of stimulated emission. If the time constant against the spontaneous decay ($\tau_{32}$) of the electron from $E_3$ to $E_2$ is negligible small in comparison to the carrier lifetime against stimulated emission ($\tau_{21}$) then there would be majority electrons already available for stimulated emission in the excited state and, hence, would result in a population inversion in the material. So, when a triggering photon of the required energy is incident on the material, the stimulated emission process would instantly take over and there would be a emission of photons from the material. This discussion suggests that, in order for the LASER
mechanism to function in an appropriate manner, a material with the above energy level system (three energy level system) is necessary.

Although, in principle, the three energy level system appears to be sufficient in producing a LASER, yet in practical scenario, the three energy level system is very inefficient. The reason of this inefficiency lies in working of this system. According to its functioning, a population inversion is created by means of transporting electrons from the ground state to an excited state higher in energy than the required excited state and then triggering photons are allowed to be incident on to the material which would cause stimulated emissions thereafter. However, one very important observation to emphasize is that, when the triggering photon is incident on the material, it has equal probability to be absorbed as well as to stimulate an emission, as already discussed earlier. The incident photon may, thus, get lost due to absorption and there would be no stimulated emission due to it. This notion holds true for every incident photon onto the material. Hence, although, the three energy level system appears promising, it has very low efficiencies due to absorption of the incident photons.

To rectify the above anomaly, a fourth energy level \( (E_0) \) is assumed which has energy lower than the ground state of the three energy level system. Hence, this energy level is called the true ground state of the electrons (as shown in figure 16.2). In order to create a population inversion, electrons are caused to migrate from this true ground state to \( E_3 \) and these electrons then undergo a fast spontaneous decay to \( E_2 \) and remain there until they jump down to \( E_1 \) by stimulated emission. After reaching \( E_1 \) these electrons again undergo a fast spontaneous decay to \( E_0 \) and
remain there until they absorb energy to jump to \(E_3\). The carrier lifetime against this spontaneous decay must be almost negligible in comparison to the carrier lifetime against stimulated emission to ensure a fast decay. In this process, the incident photon energy which causes population inversion is more than that required in the two level and the three level systems. Thus a four energy level system is better in efficiency than a three energy level system. Thus for a LASER to function appropriately in practice, a material with a four energy level system is a basic necessity. In practical LASERs, however, materials with more complicated energy level systems are used for better performance and maximum efficiencies with the available power.

Since the stimulated emission dominates over the spontaneous emission in a material when photons are incident, it is only the stimulated processes that effectively constitute the net processes in a material after it is illuminated with photons. Therefore, the net downward transition is given by:

\[
\frac{dN_2(t)}{dt} = B_{21}\rho(v)N_2 - B_{12}\rho(v)N_1
\]  

(16.9)

Since, \(B_{12}=B_{21}\), the above equation transforms to:

\[
-\frac{dN_2(t)}{dt} = B_{21}\rho(v)(N_2 - N_1)
\]  

(16.10)

This equation suggests that if \(N_2>N_1\) (which is a case of population inversion), there would be a greater number of downward transitions (stimulated emissions); whereas if \(N_2<N_1\), the number of upward transitions (absorptions) would exceed the number of downward transitions. If we assume \(N_p\) be the number of photons emitted from the material over an interval of \(t\) seconds, then:

\[
-\frac{dN_2(t)}{dt} = \frac{dN_p}{dt} = B_{21}\rho(v)(N_2 - N_1)
\]  

(16.11)

If we multiply both sides of the equation 16.11 by the energy of an emitted photon \(\hbar\nu\), we find that the quantity \(\hbar\nu N_p\) is actually the photon flux density function \(\rho(v)\) and the equation modifies to:

\[
\frac{d\rho(v)}{dt} = \hbar\nu\rho(v)B_{21}(N_2 - N_1)
\]  

(16.12)

If we substitute the value of the coefficient \(B_{21}\), calculated from equation 16.5 (where \(A_{21}\) is the reciprocal of the carrier lifetime against spontaneous emission \(\tau_{sp}\)) then the above equation may be re-written as:

\[
\frac{d\rho(v)}{dt} = \frac{c^3\hbar\nu}{8\pi m^2\nu^3\tau_{sp}} \cdot \rho(v)(N_2 - N_1)
\]  

(16.13)

The equation 16.13 shows that the rate of generation of photons with time. However, a photon is actually an electromagnetic wave travelling with the speed of light.
light as soon as it is generated. The distance travelled by the photon in time ‘t’ is, hence, equal to the distance travelled by light in the same time. In the differential form we may write this as:

$$dx = \frac{c}{n} \, dt$$  \hspace{1cm} (16.14)

Here ‘n’ is the refractive index of the medium in which the light travels. Equation 16.14 helps us to transform equation 16.13 from the time domain to the space domain which may be done by substituting the value ‘dt’ from equation 16.14 into 16.13. That is

$$\frac{d\rho(v)}{dx} \cdot \frac{c}{n} = \frac{e^{hv}}{8\pi n^3 v^3 \tau_{sp}} \cdot \rho(v) (N_2 - N_1)$$

$$\Rightarrow \frac{d\rho(v)}{dx} = \frac{hc^2}{8\pi v^2 n^2 \tau_{sp}} \cdot \rho(v) (N_2 - N_1)$$

$$\Rightarrow \frac{d\rho(v)}{dx} = G \cdot \rho(v)$$  \hspace{1cm} (16.15)

Where,

$$G = \frac{hc^2}{8\pi v^2 n^2 \tau_{sp}} \cdot (N_2 - N_1)$$

Equation 16.15 is a differential equation which can be solved by classical methods and the solution is of the form:

$$\rho(v, x) = \rho(v, x = 0) \cdot e^{Gx}$$  \hspace{1cm} (16.16)

Equation 16.16 suggests that ‘G’ be termed as a Gain constant of the system. Although there are various parameters on which the value of G depends (as seen above), the significant parameter on which the value of G depends is the term \((N_2 - N_1)\). This term, hence generates three distinct cases for the value of G which may be stated as:

**Case 1: \(N_2 < N_1\); G is Negative \((G<0)\)**

Since G is negative, the photon flux decays exponentially as it travels inside the material and so, G , in this case, may be termed as the attenuation constant.

**Case 2: \(N_2 = N_1\); G=0**

Since G=0, there is neither decay nor growth of the photon flux as it travels inside the material. Hence, this condition may be termed as the condition of transparency of the material.

**Case 3: \(N_2 > N_1\); G is Positive \((G>0)\)**
This is the case of population inversion. In this case, we see that the value of $G$ is positive and so the photon flux grows exponentially as it travels inside the material. Thus, if a condition of population inversion is maintained and the emitted photon flux is caused to travel repeatedly inside the material (before it escapes the material), it grows exponentially. Thus the goal is to make the generated radiation travel more and more inside the material while maintaining the population inversion in order to have an amplified coherent stimulated emission from the material. The above aim is actually the fundamental working principle of a LASER. In the subsequent sections, we shall discuss the way in which the above aim is achieved in a semiconductor LASER and a coherent emission is generated.