Lecture: 13
Optical Sources
The probability of existence of an electron in the conduction band of a n-type semiconductor material is given by the Fermi distribution $F(E_2)$ (as discussed in the last section). Similarly, the probability of the existence of a hole in the valence band of a p-type semiconductor material is equal to the probability of absence of an electron from the valence band and hence is the complement of Fermi-distribution $\{1-F(E_1)\}$. These two values are given as:

$$F(E_2) = e^{-(E_2-E_{Fn})/kT}$$  
$$1 - F(E_1) = e^{(E_1-E_{Fp})/kT}$$  

Here, ‘$E_{Fn}$’ and ‘$E_{Fp}$’ are the Fermi levels in the n-type and p-type semiconductors, respectively. These two levels are constant for a given amount of doping. ‘k’ and ‘T’ are Boltzmann’s constant and the absolute temperature of the semiconductor respectively.

If we look into the functions that depict the density of available energy levels in the conduction and the valence bands, we find that these functions vary very slowly with respect to energy because the energy term appears under the radical sign (Equation 12.5 and 12.6). So, if we assume a very narrow stretch of energy levels in any of the bands, the energy level density almost remains constant. Under this assumption, let us now find out the total number of photons generated by the semiconductor material. In a semiconductor material, there is no sort of priority assigned to any particular electron to recombine with a particular hole. In other words, any electron in the conduction band can combine with any hole in the valence band. The energy released in the recombination would correspond to a photon if the frequency of the radiated energy falls within the visible range of frequencies. If, for the moment, we assume the semiconductor material to be 100% efficient, the total number of photons would, hence, correspond to the total number of electrons and holes available for recombination. This quantity would be proportional to the probability of existence of an electron-hole pair in the semiconductor material which would, hence, be proportional to the product of equations 13.1 and 13.2. The total number of photons can be calculated by integration of the above product. The word ‘proportional’ is used because we have made an initial assumption that the energy level density function is almost constant over the band of energies under consideration.

$$\text{Probability of a photon} \propto F(E_2) \cdot (1 - F(E_1))$$  
$$\propto e^{-(E_2-E_{Fn})/kT} \cdot e^{(E_1-E_{Fp})/kT}$$  
$$\propto e^{-(E_2-E_{Fp})/kT}$$  
$$\propto e^{-(E_2-E_{Fp})/kT} \cdot \text{[constant]}$$  

The second term in the expression preceding equation 13.3 is a constant quantity because the two Fermi-level energies are constant for a given amount of impurity doping concentration. So, the probability of a photon generation is
proportional to the first exponential term in the equation 13.3. This term, in fact, is the exponential of the energy difference \((E_2 - E_1)\) between the combining electron and hole. This difference in energies is released as a photon in the process of recombination and hence is equal to the energy of the photon \((E_{ph}, \text{ say})\). The total number of photons, hence, generated can be obtained by integrating 13.3 within the limits \(E_c\) upto \((E_v + E_{ph})\), where \(E_c\) and \(E_v\) are the conduction and valence band energies respectively. That is,

\[
\text{Total number of Photons } \propto \int_{E_c}^{E_v + E_{ph}} e^{-\frac{(E_{ph})}{kT}} dE_2 \quad (13.4)
\]

The integrand in the above expression is a constant quantity with respect to the integral and hence can be brought out of the integral sign. This yields:

\[
\text{Total number of Photons } \propto e^{-\frac{(E_{ph})}{kT}} \left( E_v + E_{ph} - E_c \right)
\]

\[
\propto e^{-\frac{(E_{ph})}{kT}} \left( E_{ph} - (E_c - E_v) \right) \]

\[
\propto e^{-\frac{(E_{ph})}{kT}} \left( E_{ph} - E_g \right) \quad (13.5)
\]

The quantity ‘\(E_g\)’ is called the band-gap of the intrinsic semiconductor material and is a constant quantity for a given semiconductor material at a given temperature. So, the addition of an exponential term containing the band-gap as the exponent would, hence, not disturb the proportionality. The expression 13.5 can, therefore, be modified to:

\[
\text{Total number of Photons generated } \propto (E_{ph} - E_g) e^{-\frac{(E_{ph} - E_g)}{kT}} \quad (13.6)
\]

Expression 13.6 is a function that varies with the energy of the photon, which, in fact, is a function of its wavelength. (The reader should note here that, if the energy of the photon is less than the band-gap, this expression is not valid because there would occur no recombination if \((E_2 - E_1) < E_g\). So, if we plot a curve between the total number of photons versus the photon energies, we would, hence, obtain the spectral density of the source. More the number of generated photons more is the output power. Hence, the above curve is plotted between source power and the photon energy which would have the same shape as the above curve and would give the energy spectral density of the source. Thus the spectral distribution of a Light Emitting Diode (LED) is shown below.

![Figure 13.1: Spectral Distribution of an LED](image-url)
As seen from the curve, the output power of the semiconductor source is zero if the energy of the photon is equal to the band-gap of the semiconductor material. In other words, there is no photon emitted which has an energy equal to the band-gap of the material. As the energy of the photon increases, the 1\textsuperscript{st} term in expression 13.6 dominates and the curve has an increasing trend. But this increase occurs only upto certain energy, after which, the exponential term starts dominating the behaviour of the curve and it starts a downward trend. The value of the photon energy at the position of maxima is $E_g + kT$ (eV). This means maximum number of photons emitted, have energies equal to $E_g + kT$ (eV). If the wavelength corresponding to this energy is calculated, then that is the wavelength of light emitted by the optical source. Different semiconductor materials have different band-gaps and consequently, the wavelength of the output also differs.

If the width of the curve shown in figure 13.1 is calculated, it is found to be about 2$kT$, which is independent of the band-gap of the material used. Thus, although different materials have different centre wavelengths, as long as they are operated in the same operating conditions of temperature, the spectral widths of all those sources would be the same and cannot be controlled. This feature is greatly undesirable because, the spectral width of the source is, hence, not in our control but depends purely on the operating conditions of the source. However, in the presence on impurities in the material, the spectral widths may vary from about 1.5$kT$ to 3.5$kT$. The expression for a quantitative calculation of the spectral width of the source can be derived as follows:

From equation 12.4, we may write,

$$\lambda = \frac{hc}{E_{ph}} \quad (13.6)$$

Now, by applying the difference operator ‘$\Delta$’ on both sides of equation 13.6, we find that the spectral width of the source is given as:

$$\frac{\Delta \lambda}{\lambda} = \frac{\Delta E_{ph}}{E_{ph}} = \frac{2kT}{E_{ph}} \quad (13.7)$$

Using the above relation, the spectral widths of different types of materials used for making optical sources can be calculated. At room temperature, the spectral widths of the sources corresponding to the three windows of optical communication can be tabulated as follows:

**Table 13.1:** Spectral Widths of different sources

<table>
<thead>
<tr>
<th>$\lambda$ (nm)</th>
<th>$\Delta \lambda$ (nm)</th>
<th>$\Delta \lambda/\lambda$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>850</td>
<td>30</td>
<td>3.6</td>
</tr>
<tr>
<td>1310</td>
<td>70</td>
<td>5.5</td>
</tr>
<tr>
<td>1550</td>
<td>100</td>
<td>6.5</td>
</tr>
</tbody>
</table>
From the above table it is clear that as wavelength of the light emitted by the source increases, its spectral width also increases and so does the dispersion on the optical fiber. This may seem contradictory to our earlier discussion, where we deduced that 1310 nm and 1550nm windows were low-loss and low-dispersive windows as compared to 850 nm. However, one may note that, in the earlier discussion, we deduced the above conclusion assuming all the other factors of the optical fiber remaining constant. Also, one can note that this increase in the dispersion of the fiber is not because of the fiber characteristics but because of the optical sources used for the application. Thus simple semiconductor LEDs show a high value of dispersion with increasing wavelength of operation and hence have a very low data rate when used as a source in the optical system.

Another important observation about an LED spectral distribution of figure 13.1 is that, although the theoretical spectral distribution looks highly asymmetric as shown in the above figure, but in practice the spectral distribution looks like the one shown in the following figure 13.2. The practical distribution looks almost symmetric about the central maxima. There occurs no sharp cut-off of radiation at Eg, rather there is a smooth variation on both sides of the central emission.

![Figure 13.2: Spectral Distribution of Practical LEDs](image)

The reason behind the non-existence of cut-off at Eg lies in the energy band diagram of a practical LED. In our discussion, we argued that, there would be no recombination for energies less than the band-gap energy because there are no energy levels in the forbidden band-gap to accommodate charge carriers. Hence, the minimum radiated energy would be equal to the band-gap energy and not below that. However, in a practical LED, due to the presence of impurities, few energy levels exist in the forbidden band too. The recombinations between such energy levels would then correspond to energies less than Eg, which in theory is a point of cut-off of radiation.

We have already discussed that there are two types of electron-hole recombinations possible in a semiconductor material- radiative and non-radiative recombinations. Of these two recombinations, radiative recombination is important to us in context of optical sources. So, a parameter can be defined that indicates the number of radiative recombinations out of the total number of recombinations in the
semiconductor material. This quantity is called the quantum efficiency of the semiconductor material. More the quantum efficiency better is the semiconductor material. There are two types of quantum efficiencies that are defined for an optical source- internal and external quantum efficiencies. Internal quantum efficiency is related to the intrinsic nature of the semiconductor source. External quantum efficiency is related to the light guiding efficiency of the source into the optical fiber. Internal and external quantum efficiencies are defined below:

**Internal Quantum Efficiency:**

\[
\eta_{\text{int}} = \frac{\text{Number of Photons generated}}{\text{Total Number of Electron–hole recombinations}} \tag{13.8}
\]

**External Quantum Efficiency:**

\[
\eta_{\text{ext}} = \frac{\text{Number of Photons guided to the optical fiber}}{\text{Total number of photons generated}} \tag{13.9}
\]

Internal quantum efficiency is dependent on the intrinsic properties of the material and the manufacturing process of the semiconductor. External quantum efficiency, on the other hand, signifies the number of photons that satisfy the requirements to be launched into the optical fiber. The total efficiency is the product of the internal and external quantum efficiencies.

The rate of electron-hole recombination in a semiconductor material is proportional to the carrier density in the semiconductor material. That is, if \( N \) is the carrier density of a semiconductor material at any time then:

\[
\frac{\partial N}{\partial t} \propto N \tag{13.10}
\]

Inequality (13.10) forms a simple differential equation, which can be solved to yield:

\[
N(t) = N_0 e^{-t/\tau} \tag{13.11}
\]

Differentiating equation 13.11 we have:

\[
- \frac{\partial N_0(t)}{\partial t} = \frac{N(t)}{\tau} \tag{13.12}
\]

Here, \( N_0 \) is the initial charge carrier density in the semiconductor material and \( \tau \) is the average life-time of a carrier against recombination. Since there are two types of recombinations, let us assume \( \tau_r \) be the life-time of a carrier against radiative recombination and \( \tau_{nr} \) be the life-time of a carrier against non-radiative recombination. The rate of recombination given by equation 13.12 is the total rate of
recombination which is the sum total of the radiative and the non-radiative recombination processes. So, the total rate of recombination can be written as:

\[
\frac{N(t)}{\tau} = \frac{N(t)}{\tau_{rr}} + \frac{N(t)}{\tau_{nr}}
\]

or,

\[
\frac{1}{\tau} = \frac{1}{\tau_{rr}} + \frac{1}{\tau_{nr}}
\]

(13.13)

From the above equation it is clear that the total effective carrier life-time will be determined by the smaller of the two life-times viz. ‘\(\tau_{rr}\)’ and ‘\(\tau_{nr}\)’. The internal and external quantum efficiencies can now be calculated by substituting the values of the terms given in their definition. Let us first derive the internal quantum efficiency.

\[
\eta_{int} = \frac{\frac{\partial N(t)}{\partial t}}{\frac{\partial N(t)}{\partial t}} - \frac{\tau_{rr}}{\tau_{nr}}
\]

(13.14)

If ‘\(\tau_{nr}\)’ \(>>\) ‘\(\tau_{rr}\)’, then the internal quantum efficiency is almost equal to unity (100%). So for a good optical source the carrier life-time against non-radiative recombination must be very large as compared to the carrier life-time against radiative recombination. The internal quantum efficiency value for GaAs is about 50%.

To calculate the second and more important and complicated type of efficiency (external quantum efficiency), let us look into the various stages that a generated photon has to go through before it can be guided into the optical fiber.

Figure 13.3: Photon Generation in a Semiconductor material.

In a semiconductor material, radiative recombinations do not occur at the surface of the semiconductor material; rather it occurs deep inside the material. Since the generated photon does not have any history, it has equal probability of
travelling in any direction in 3D space and as a result has equal probability of being incident on the semiconductor boundary at any arbitrary angle of incidence. Incidentally, a semiconductor material has a very high value of refractive index which is usually higher than its surrounding medium which, most of the times, is air. So if the angle incidence of the photon at the semiconductor-air boundary is greater than the critical angle of the semiconductor material, the photon is totally internally reflected back into the semiconductor material. If the photon is incident at the critical angle, the photon then travels along the semiconductor boundary. For any angle of incidence below the critical angle, the photon is emitted out from the surface of the semiconductor material at the depletion region. So for a photon, there exists a cone of emission which has an angle twice the critical angle. Any incident ray lying outside this cone is reflected back into the semiconductor. This situation is depicted in the figure 13.4 below.

![Figure 13.4: Photon emission from Semiconductor](image)

What we are trying to say here is that, although the generated photon has equal probability to travel in all possible directions, there are only certain conical regions on the surface of the semiconductor material through which it can be emitted out. The angle of this emission cone is given by:

\[ \theta_c = \sin^{-1} \left( \frac{n_s}{n_a} \right) \] (13.15)

For a semiconductor material like GaAs (refractive index=3.6) in air, the angle of the emission cone is given as:

\[ \theta_{GaAs} \approx \sin^{-1} \left( \frac{1}{3.6} \right) \approx 18^0 \text{ to } 20^0 \]

The above discussion signifies that, although photons are generated with equal directional probabilities, the photons lying in the emission cone only are actually emitted out and the rest are not. If we find out this fraction of photons out of the total number of photons generated, the result would give the external quantum efficiency of the semiconductor material. An expression for the external quantum efficiency can be, hence, derived and is found to be:

\[ \eta_{\text{ext}} = \frac{n_a^2}{4n_s^2} \] (13.15)
For GaAs, this value is about 0.0193, which means that out of every 100 photons generated in GaAs, only about 1-2 photons are emitted out of the specimen. This factor thus accounts to a large loss in efficiency of the semiconductor sample, although it may have large internal quantum efficiency. This suggests the fact that, generation of photons seems to be a comparatively easier task than to make the generated photon available for launching into the optical fiber.