Session 45

Tutorial - Frequency Domain Analysis of Two Band Filter Bank
LPF (cutoff $\frac{2\pi}{3}$) -> \( Y_1 \) -> \( \sqrt{2} \) -> \( Y_3 \) -> \( A \)

HPF (cutoff $\frac{2\pi}{3}$) -> \( Y_2 \) -> \( \sqrt{2} \) -> \( \sqrt{2} \) -> \( Y_4 \) -> \( B \)

Analysis side
Sinusoidal
frequency domain

Discrete Time Fourier
Transforms (DTFT &
"Prototype"

Zero phase $X(e^{j\omega})$
In general, it is always a good idea to consider three successive periods of 

$2\pi: [-3\pi, -\pi], [-\pi, +\pi], [+\pi, +3\pi]$
$Y_2$ $Y_3$

$\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}$

$-\frac{8\pi}{3} \quad -\frac{4\pi}{3} \quad -\frac{2\pi}{3} \quad \frac{2\pi}{3} \quad \frac{4\pi}{3} \quad \frac{8\pi}{3}$

$-3\pi \quad -2\pi \quad -\pi \quad 0 \quad \pi \quad 2\pi \quad 3\pi$
Z domain

$g[m]\xrightarrow{\sqrt{2}}{\uparrow{2}}\xrightarrow{\lceil{2}}{g[m]}$

$G_m(z)$

$G_{m[x]}(z)$
\[ G_{\text{out}}(z) = \frac{1}{2} \left\{ G_{\text{min}}(z) + G_{\text{min}}(-z) \right\} \]

\[ z \rightarrow e^{j\omega} \]
\[
\text{Graph}(E_{\text{out}}) = \frac{1}{2} \left\{ \text{Graph}(E_{\text{in}}) \uparrow \uparrow \text{Graph}(E_{\text{in}}) \right\} + \text{Graph}(E_{\text{in}}(\kappa \pm \pi))
\]
\[ g_{\text{out}}(z) = \frac{1}{2} \text{original DTFT} + \frac{1}{2} \text{aliased DTFT} \]
\[ e^{jw} - e^{jw} = e^{j\pi jw} = e^{j(w+\pi)} = e \]
\[ \gamma_2 \rightarrow \downarrow 2 \rightarrow \uparrow 2 \rightarrow \gamma_2 \]

\[ \gamma(e^{j\omega}) = \frac{1}{6} \{ \gamma_2(e^{j\omega}) + \gamma_2(e^{j\omega}) \} \]

\[ j(f(t+1)) \]
HPF with cutoff $2\pi$
retains $\frac{1}{2} \log(2)$;
and destroys $\frac{1}{2} \log(\text{aliases}).$
\[ Y_1 \rightarrow \sqrt{2} \rightarrow 12 \rightarrow Y_5 \]

\[ Y_5(e^{j\omega}) = \frac{1}{2} Y_1(e^{j\omega}) + \frac{1}{2} Y_1(e^{j(2\pi + \omega)}) \]

original alias.
aliases + original

-3\pi, -2\pi, -2\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, 2\pi, 3\pi, 4\pi, \frac{8\pi}{3}, \frac{11\pi}{3}, \frac{17\pi}{3}, \frac{22\pi}{3}
Subjecting to LFF

\[ \frac{1}{3} \frac{1}{3} \frac{1}{3} \]

-1 3 11

-1 \frac{1}{3} 0 \frac{1}{3} \frac{1}{3} 11
Aliasing has occurred in a band of extent \( \frac{\pi}{3} - \frac{\pi}{2} \) = \( \frac{\pi}{6} \) on either side of \( \frac{\pi}{2} \).
Perhaps we could interpret as: \[ \sqrt{\frac{3}{2}} \]
LPF cutoff \( \frac{3}{2} \)

Is this possible?
Perhaps it may be better to do

\[ \text{no loss of information} \quad \text{incur loss only in the end!} \]
Prototype: $X(e^{j\omega})$
filtered \[ x = \frac{2}{3} \]

-11 - \frac{2\pi}{3} \rightarrow -\frac{\pi}{3} \rightarrow \omega \cdot \frac{2\pi}{3} \rightarrow \pi
Filtered \( X(e^j\omega) \)

\[ X \rightarrow^2 \rightarrow Y_1(e^j\omega) \]

\[ Y_1(e^j\omega) \rightarrow \text{filter} \]

\[ F_1 \]

\[ Y_2(e^j\omega) \]

\[ Y_2(e^j\omega) \rightarrow V_3 \]

\[ Y_3 \rightarrow 0 \]
F_i needs to be an ideal lowpass filter cutoff $\frac{f_1}{2}$