Self Evaluation Quizzes

Q 1. Obtain the Z-Transform of a general down-sampler (down-sampling by a factor of M).

Ans. Down-sampler can be broken down into two steps as modulation by a sequence and then inverse up-sampling. General sequence for modulation in case of down-sampling by a factor of M can be written as:

$$\frac{1}{M} \sum_{l=0}^{M-1} e^{-j\frac{2\pi l}{M}}$$

(1)

Using this equation, we can write the Z-Transform of down sampler by factor M as:

$$X_{out,d}(Z) = \frac{1}{M} \sum_{l=0}^{M-1} X_{in}(Z^{\frac{1}{M}} * e^{-j\frac{2\pi l}{M}})$$

(2)

Q 2. Obtain the output of M-channel filter bank.

Ans. Z-Transform of M-channel filter bank which has M analysis filters denoted by $H_0(Z)$, $H_1(Z)$, \ldots $H_{M-1}(Z)$ and corresponding M synthesis filter denoted by $F_0(Z)$, $F_1(Z)$ \ldots $F_{M-1}(Z)$ can be written as follows:

$$X_{out,d}(Z) = \frac{1}{M} \sum_{l=0}^{M-1} (X_{in}(Z * e^{-j\frac{2\pi l}{M}}) \sum_{k=0}^{M-1} (H_k(Z * e^{-j\frac{2\pi l}{M}})) * F_k(Z))$$

(3)

(For more reading on filter banks, you can refer “Multi-rate Systems and Filter Banks” by P.P.Vaidyanathan).