STiCM Lecture 11: Unit 3
Physical Quantities – scalars, vectors....
We must examine how their ‘components’ transform under the rotation of a coordinate frame of reference.

Unit 3: Polar Coordinates

Learning goals

‘symmetry’

Learn to use an appropriate coordinate system to simplify analysis.

Goals: Physical quantities are tensors of various ranks.

We must examine how their ‘components’ transform under the rotation of a coordinate frame of reference.
The sun and the planets were considered to move on a small circle (called ‘epicycle’) whose center would move on a large circle (called ‘deferent’).

Claudius Ptolemaeus (AD100-170) (called Ptolemy) worked in the library of Alexandria.
Contributions of Indian Astronomers to the Understanding of Heliocentric Coordinate System

Aryabhata (b. 476 A.D.) - ‘ARYABHATYA’ (499 A.D.)

Bhaskara I (A.D. 600) - ‘MAHABHASKARIYA’,
‘LAGHUBHASKARIYA’, ‘ARYABHATIYA BHASHYA’

Brahmagupta (A.D. 591) - ‘BRAMA SIDDHANTA’

Vateshwa (A.D. 880) - ‘VATESHWARA SIDDHANTA’

Manjulacharya-(A.D. 932) - ‘LAGHUMANASA’
[ Dealt with Precession of equinoxes ]

Aryabhata II (A.D. 950) - ‘MAHASIDDHANTA’

Bhaskaracharya II (A.D. 1114) ‘SIDDHANTA SHIROMANI’ [ This work contains many formulas from spherical trigonometry ]………etc.
Modification of the earlier Indian planetary theory by the Kerala astronomers (c. 1500 AD) and the implied heliocentric picture of planetary motion

K. Ramasubramanian, M. D. Srinivas and M. S. Sriram

From Golapaada, by Aryabhata, ~500 AD

Just as a man in a boat moving sees the stationary objects (on either side of the river) as moving backward, so are the stationary stars seen by the people at Lanka (i.e. reference coordinate on the equator) as moving exactly toward the west.

Nicolus Copurnicus
1473-1543
Despite admitting the advantages of the heliocentric coordinate system, Descartes was reluctant to promote the “certain and evident proof” in favor of the heliocentric system since it was against the will of the church.
April 1633: Galileo is interrogated before the Inquisition.

June:
Galileo sentenced to prison for an indefinite term.

December: Galileo is allowed to return to his villa in Florence, where he lived under house-arrest.

1992: Catholic Church formally admits that Galileo's views on the solar system are correct.

http://www.law.umkc.edu/faculty/projects/ftrials/galileo/galileochronology.html
Definition of a vector: “magnitude” and “direction”

Is rotation by 90 degrees a vector?
How do vectors transform under rotation of a coordinate system?

\[ \vec{V} = V_x \hat{e}_x + V_y \hat{e}_y \]

Same vector can also be written as

\[ \vec{V} = V_{x'} \hat{e}_{x'} + V_{y'} \hat{e}_{y'} \]

This can be generalized into three (or N) dimensions.
\[ \{V_x, V_y, V_z\} \]

\[ \{V_x', V_y', V_z'\} \]
\[ V_{x'} = V_x \left[ \hat{e}_{x'} \cdot \hat{e}_x \right] + V_y \left[ \hat{e}_{x'} \cdot \hat{e}_y \right] + V_z \left[ \hat{e}_{x'} \cdot \hat{e}_z \right] \]

\[ V_{y'} = V_x \left[ \hat{e}_{y'} \cdot \hat{e}_x \right] + V_y \left[ \hat{e}_{y'} \cdot \hat{e}_y \right] + V_z \left[ \hat{e}_{y'} \cdot \hat{e}_z \right] \]

\[ V_{z'} = V_x \left[ \hat{e}_{z'} \cdot \hat{e}_x \right] + V_y \left[ \hat{e}_{z'} \cdot \hat{e}_y \right] + V_z \left[ \hat{e}_{z'} \cdot \hat{e}_z \right] \]

Compact matrix form

\[
\begin{bmatrix}
V_{x'} \\
V_{y'} \\
V_{z'}
\end{bmatrix} =
\begin{bmatrix}
\hat{e}_{x'} \cdot \hat{e}_x & \hat{e}_{x'} \cdot \hat{e}_y & \hat{e}_{x'} \cdot \hat{e}_z \\
\hat{e}_{y'} \cdot \hat{e}_x & \hat{e}_{y'} \cdot \hat{e}_y & \hat{e}_{y'} \cdot \hat{e}_z \\
\hat{e}_{z'} \cdot \hat{e}_x & \hat{e}_{z'} \cdot \hat{e}_y & \hat{e}_{z'} \cdot \hat{e}_z
\end{bmatrix}
\begin{bmatrix}
V_x \\
V_y \\
V_z
\end{bmatrix}
\]
\[
\begin{bmatrix}
    x' \\
y' \\
z'
\end{bmatrix} = 
\begin{bmatrix}
    R_{xx} & R_{xy} & R_{xz} \\
    R_{yx} & R_{yy} & R_{yz} \\
    R_{zx} & R_{zy} & R_{zz}
\end{bmatrix}
\begin{bmatrix}
    x \\
y \\
z
\end{bmatrix}
\]

\[
\begin{vmatrix}
    R_{xx} & R_{xy} & R_{xz} \\
    R_{yx} & R_{yy} & R_{yz} \\
    R_{zx} & R_{zy} & R_{zz}
\end{vmatrix} = \pm 1 \quad \vec{r}_R = \mathbf{R}\vec{r}; \quad |\mathbf{R}| = \pm 1
\]

**ROTATION:** $|\mathbf{R}| = +1$  \hspace{1cm} **PARITY / INVERSION:** $|\mathbf{R}| = -1$
REFLECTION

\[
\begin{pmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} =
\begin{pmatrix}
-x \\
y \\
z
\end{pmatrix}
\]

LEFT  \leftrightarrow  RIGHT

TOP  \leftrightarrow  BOTTOM

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} =
\begin{pmatrix}
x \\
y \\
-z
\end{pmatrix}
\]

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Too much Mathematics?

“IF YOU WANT TO READ THE BOOK OF THE UNIVERSE,

YOU MUST KNOW ITS LANGUAGE, WHICH IS MATHEMATICS”.

Who said that?

Grandpa of Engineering?

Father of experimental Physics!
\[ \vec{C} = \vec{A} \times \vec{B} \]

Angular momentum

\[ \vec{l} = \vec{r} \times \vec{p} \]

Right-hand cross product

\[ \vec{r} \times \vec{p} \]
\[ \vec{C} = \vec{A} \times \vec{B} \]

\[ \vec{l} = \vec{r} \times \vec{p} \]

**angular momentum**

\[ \vec{r}_\text{right-hand-cross-product} = \vec{r}_{\text{image}} \times \vec{p}_{\text{image}} \]

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Axial vector (pseudo vector) does not transform like a position vector under reflection.

Its components are governed by a different transformation law with respect to rotation of the coordinate system.
Examples:
Some ‘real physical quantities’

Angular Momentum Vector $\vec{r} \times \vec{p}$

Force on a charged particle moving in an electromagnetic field

$$\vec{F} = q \left\{ \vec{E} + \vec{v} \times \vec{B} \right\}$$  \hspace{1cm} \text{Lorentz Force}

The Lorentz force, like any other force, is a polar vector, since it includes the cross-product of a polar vector $\vec{v}$ with a pseudo-vector $\vec{B}$. 
Algebra of Pseudo Vectors and Examples

**Dot and cross products:**
- Polar x Polar = Axial
- Polar x Axial = Polar
- Axial x Axial = Axial
- Axial • Polar = Pseudo-scalar

**Examples for axial (pseudo) vectors:**

- Torque \( \mathbf{\tau} = \mathbf{r} \times \mathbf{f} \)
- Angular Momentum \( \mathbf{L} = \mathbf{r} \times \mathbf{p} \)
- Magnetic field \( \mathbf{F}_{\text{mag.}} = q\mathbf{\hat{v}} \times \mathbf{B} \)

**Important:** An axial vector can never be equated with a polar vector
We have learned that physical quantities are represented by scalars, vectors, tensors etc.

Scalars: tensors of rank zero

Vectors: tensors of rank one

Scalars / pseudo-scalars

Vectors / pseudo-vectors

Polar vectors / Axial vectors
WE WILL TAKE A BREAK...

...... ANY QUESTIONS?

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Next: vectors in Polar coordinates
STiCM

SELECT / SPECIAL TOPICS IN CLASSICAL MECHANICS

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STiCM Lecture 12: Unit 3

Plane Polar Coordinates
Cylindrical Polar Coordinates
Spherical Polar Coordinates
MOTORCYCLE MANIA

THE TORRES BROTHERS!

FIRST 5, THEN 7 GUYS RACE THEIR BIKES INSIDE A 16 FOOT STEEL GLOBE.

UNBELIEVABLE!

http://myspace.vtap.com/video/Motor+Cycle+Mania/CL0177433717_7cf78882_V0ILSTE1MTlxN35pbozfnnE6YnJ-Ync6V0ILSTE1MTlxNyxDTDAwNzl4MDMzMl-aW46Nn5xOnJs

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\[ x = \rho \cos \varphi \]
\[ y = \rho \sin \varphi \]
\[-\infty < x < \infty \]
\[-\infty < y < \infty \]

\[ \rho = +\sqrt{x^2 + y^2} \]
\[ \varphi = \tan^{-1}\left(\frac{y}{x}\right) \]
\[ \rho : 0 \leq \rho < \infty \]
\[ \varphi : 0 \leq \varphi < 2\pi \]

\[ \hat{e}_\rho = \cos \varphi \hat{e}_x + \sin \varphi \hat{e}_y \]
\[ \hat{e}_\varphi = -\sin \varphi \hat{e}_x + \cos \varphi \hat{e}_y \]

\[ \hat{e}_x = \cos \varphi \hat{e}_\rho - \sin \varphi \hat{e}_\varphi \]
\[ \hat{e}_y = \sin \varphi \hat{e}_\rho + \cos \varphi \hat{e}_\varphi \]

\[ \hat{e}_\rho \cdot \hat{e}_\rho = 1 = \hat{e}_\varphi \cdot \hat{e}_\varphi \]
\[ \hat{e}_\rho \cdot \hat{e}_\varphi = 0 \]

\((\hat{e}_\rho, \hat{e}_\varphi)\) constitute an orthogonal pair of base vectors
Position vector

\[ \vec{\rho} = \rho \hat{e}_\rho \]

VELOCITY?

ACCELERATION?

Note that \((\hat{e}_\rho, \hat{e}_\phi)\) are not constant vectors.

To get acceleration, we have to do that twice!

\[ \frac{d}{dt} \left[ \text{Product of two functions} \right] \]

\[ \frac{d}{dt} \frac{d}{dt} \]
\((\hat{e}_\rho, \hat{e}_\varphi)\) are not constant vectors, whereas \((\hat{e}_x, \hat{e}_y)\) are.

\[
\hat{e}_\rho = \cos \varphi \hat{e}_x + \sin \varphi \hat{e}_y
\]

\[
\hat{e}_\varphi = -\sin \varphi \hat{e}_x + \cos \varphi \hat{e}_y
\]
\[ \hat{e}_\rho = \hat{e}_\rho(\rho, \varphi) \]

\[ \hat{e}_\varphi = \hat{e}_\varphi(\rho, \varphi) \]

How do these unit vectors change with the azimuthal angle?

\[ \hat{e}_\rho = \cos \varphi \hat{e}_x + \sin \varphi \hat{e}_y \]

\[ \frac{\partial \hat{e}_\rho}{\partial \varphi} = -\sin \varphi \hat{e}_x + \cos \varphi \hat{e}_y \]

\[ \hat{e}_x = \cos \varphi \hat{e}_\rho - \sin \varphi \hat{e}_\varphi \]

\[ \hat{e}_y = \sin \varphi \hat{e}_\rho + \cos \varphi \hat{e}_\varphi \]

\[ \frac{\partial \hat{e}_\rho}{\partial \varphi} = -\sin \varphi \left( \cos \varphi \hat{e}_\rho - \sin \varphi \hat{e}_\varphi \right) + \cos \varphi \left( \sin \varphi \hat{e}_\rho + \cos \varphi \hat{e}_\varphi \right) \]

\[ = \hat{e}_\varphi \]
Geometrical determination of $\frac{\partial \hat{e}_\rho}{\partial \varphi}$

Consider $(\hat{e}_\rho, \hat{e}_\varphi)$ at two neighboring points, infinitesimally close to each other.

"Unit Circle"

$$
\hat{e}_{\rho_2} - \hat{e}_{\rho_1} = \delta \varphi \hat{e}_\varphi
$$

$$
\lim_{\delta \varphi \to 0} \frac{\hat{e}_{\rho_2} - \hat{e}_{\rho_1}}{\delta \varphi} = \lim_{\delta \varphi \to 0} \frac{\delta \hat{e}_\rho}{\delta \varphi} = \frac{\partial \hat{e}_\rho}{\partial \varphi} = \hat{e}_\varphi
$$
\((\hat{e}_\rho, \hat{e}_\varphi)\) are not constant vectors.

\[ \hat{e}_\rho = \cos \varphi \hat{e}_x + \sin \varphi \hat{e}_y \]
\[ \hat{e}_\varphi = -\sin \varphi \hat{e}_x + \cos \varphi \hat{e}_y \]

\[ \lim_{\delta \varphi \to 0} \frac{\hat{e}_\varphi - \hat{e}_\varphi^1}{\delta \varphi} = \lim_{\delta \varphi \to 0} \frac{\delta \hat{e}_\varphi}{\delta \varphi} = \frac{\partial \hat{e}_\varphi}{\partial \varphi} = -\hat{e}_\rho \]

\[ \frac{\partial \hat{e}_\rho}{\partial \rho} = 0 \]
\[ \frac{\partial \hat{e}_\varphi}{\partial \rho} = \hat{e}_\varphi \]
\[ \frac{\partial \hat{e}_\rho}{\partial \varphi} = 0 \]
\[ \frac{\partial \hat{e}_\varphi}{\partial \varphi} = -\hat{e}_\rho \]
If \( \xi = \xi(u) \) and \( u = \phi(x) \),
then \( \frac{d\xi}{dx} \) will be a measure of the sensitivity of \( \xi \) to changes in \( x \):
\[
\frac{d\xi}{dx} = \left( \frac{d\xi}{du} \right) \left( \frac{du}{dx} \right)
\]

If \( \xi = \xi(u, v) \)
where \( u = u(x), v = v(x) \),
the rate at which \( \xi \) will change with respect to \( x \)
will be given by:
\[
\frac{d\xi}{dx} = \left( \frac{\partial \xi}{\partial u} \right) \left( \frac{du}{dx} \right) + \left( \frac{\partial \xi}{\partial v} \right) \left( \frac{dv}{dx} \right)
\]

If \( \xi = \xi(u, v, x) \) where \( u = u(x), v = v(x) \),
the rate at which \( \xi \) will change with respect to \( x \) will be given by:
\[
\frac{d\xi}{dx} = \left( \frac{\partial \xi}{\partial u} \right) \left( \frac{du}{dx} \right) + \left( \frac{\partial \xi}{\partial v} \right) \left( \frac{dv}{dx} \right) + \left( \frac{\partial \xi}{\partial x} \right).\]
Elemental area in plane polar coordinates

\[ dA = \rho d\rho d\phi \]

\[
\int_0^R \int_0^{2\pi} \rho d\rho d\phi = \frac{R^2}{2} 2\pi = \pi R^2
\]

Position vector & Velocity in plane polar coordinates

\[ \overrightarrow{\rho} = \rho \hat{\rho} \]
\[ \overrightarrow{d\rho} = (d\rho) \hat{\rho} + \rho d\hat{\rho}_\rho \]
\[ \overrightarrow{v} = \dot{\rho} = \frac{d\overrightarrow{\rho}}{dt} = \frac{d}{dt} (\rho \hat{\rho}_\rho) \]
\[ = \frac{d\rho}{dt} \hat{\rho}_\rho + \rho \frac{d\hat{\rho}_\rho}{dt} \]

\[
\frac{\partial \hat{\rho}_\rho}{\partial \rho} = 0, \quad \frac{\partial \hat{\rho}_\rho}{\partial \phi} = \hat{\phi}, \\
\frac{\partial \hat{\phi}}{\partial \rho} = 0, \quad \frac{\partial \hat{\phi}}{\partial \phi} = -\hat{\rho}
\]
Motion of a particle in plane polar coordinates

\[
\frac{d\hat{e}_\rho}{dt} = \frac{\partial \hat{e}_\rho}{\partial \varphi} \dot{\varphi} = \hat{e}_\varphi \dot{\varphi},
\]

\[
\frac{d\hat{e}_\varphi}{dt} = \frac{\partial \hat{e}_\varphi}{\partial \varphi} \dot{\varphi} = -\hat{e}_\rho \dot{\varphi}
\]

\[
\vec{v} = \dot{\rho} = \frac{d\rho}{dt} = \frac{d}{dt} \left( \rho \hat{e}_\rho \right) = \frac{d\rho}{dt} \hat{e}_\rho + \rho \frac{d\hat{e}_\rho}{dt} = \dot{\rho} \hat{e}_\rho + \rho \dot{\varphi} \hat{e}_\varphi
\]

Radial velocity and Azimuthal velocity
\[ \vec{v} = \dot{\rho} \hat{e}_\rho + \rho \dot{\phi} \hat{e}_\phi \]  

instantaneous velocity

\[
\frac{d\hat{e}_\rho}{dt} = \frac{\partial \hat{e}_\rho}{\partial \phi} \dot{\phi} = \hat{e}_\phi \dot{\phi}
\]

and

\[
\frac{d\hat{e}_\phi}{dt} = \frac{\partial \hat{e}_\phi}{\partial \phi} \dot{\phi} = -\hat{e}_\rho \dot{\phi}
\]

acceleration

\[
\vec{a} = \frac{d\vec{v}}{dt} = \ddot{\rho} \hat{e}_\rho + \dot{\rho} \frac{d\hat{e}_\rho}{dt} + \dot{\rho} \phi \hat{e}_\phi + \rho \ddot{\phi} \hat{e}_\phi + \rho \dot{\phi} \frac{d\hat{e}_\phi}{dt}
\]

\[
\Rightarrow \vec{a} = (\ddot{\rho} - \rho \dot{\phi}^2) \hat{e}_\rho + (2\dot{\rho} \dot{\phi} + \rho \ddot{\phi}) \hat{e}_\phi
\]

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Cylindrical Polar Coordinates

\[ \hat{e}_x, \hat{e}_\phi, \hat{e}_z \]

Spherical Polar Coordinates

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\[ r = \sqrt{x^2 + y^2 + z^2} \]
\[ \tan \theta = \frac{\rho}{z} = \frac{\sqrt{x^2 + y^2}}{z} \]
\[ \theta = \tan^{-1} \left( \frac{\sqrt{x^2 + y^2}}{z} \right) \]
\[ \varphi = \tan^{-1} \left( \frac{y}{x} \right) \]
\[ x = r \sin \theta \cos \varphi \]
\[ y = r \sin \theta \sin \varphi \]
\[ z = r \cos \theta \]
Transformations of the Unit Vectors

\[
\begin{bmatrix}
\hat{e}_r \\
\hat{e}_\theta \\
\hat{e}_\varphi
\end{bmatrix} =
\begin{bmatrix}
sin \theta \cos \varphi & sin \theta \sin \varphi & \cos \theta \\
\cos \theta \cos \varphi & \cos \theta \sin \varphi & -\sin \theta \\
-\sin \varphi & \cos \varphi & 0
\end{bmatrix}
\begin{bmatrix}
\hat{e}_x \\
\hat{e}_y \\
\hat{e}_z
\end{bmatrix}
\]

Get the inverse matrix, and write the inverse transformations.

\[
\begin{bmatrix}
\hat{e}_x \\
\hat{e}_y \\
\hat{e}_z
\end{bmatrix} =
\begin{bmatrix}
sin \theta \cos \varphi & \cos \theta \cos \varphi & -\sin \varphi \\
\sin \theta \sin \varphi & \cos \theta \sin \varphi & \cos \varphi \\
\cos \theta & -\sin \theta & 0
\end{bmatrix}
\begin{bmatrix}
\hat{e}_r \\
\hat{e}_\theta \\
\hat{e}_\varphi
\end{bmatrix}
\]
The point is displaced to a new point on the sphere of same radius and in the same plane.

\[ r = \text{constant} \quad \phi = \text{constant} \]

Recognize the distance between the old position and the new position to be

\[ r \, d\theta \]

\[ \rho = r \sin \theta \]
The point is displaced to a new point on the sphere \( r = \text{constant} \) and on the surface of the inverted cone \( \theta = \text{constant} \).

The distance between the old position and the new position is given by

\[
\rho d\varphi = r \sin \theta d\varphi
\]

Recognize the distance between the old position and the new position to be

\[
\rho d\varphi = r \sin \theta d\varphi
\]
Volume spanned by the three displacements through:

\[ dV = (dr)(rd\theta)(r \sin \theta d\phi) = r^2 \sin \theta \, dr \, d\theta \, d\phi \]
In the limit
\[ \delta \theta \rightarrow 0, \]
\[ \frac{\partial \hat{e}_r}{\partial \theta} = \hat{e}_\theta \]

The point is displaced to a new point on the sphere of same radius and in the same plane
\[ r = \text{constant} \]

Distance between the 1\textsuperscript{st} position and 2\textsuperscript{nd} position is \[ rd\theta \]
Partial derivatives of the unit vectors with respect to the coordinates:

\[ \frac{\partial \hat{e}_r}{\partial r} = \frac{\partial \hat{e}_\theta}{\partial r} = \frac{\partial \hat{e}_\phi}{\partial r} = 0 \]

If imagining complicated geometrical three-dimensional objects is getting difficult, you can use the ‘chain rule’ of taking derivatives to get the partial derivatives of the unit vectors using these transformation rules, as illustrated on the next page.

\[ \frac{\partial \hat{e}_r}{\partial \theta} = \hat{e}_\theta \]
\[ \frac{\partial \hat{e}_\theta}{\partial \theta} = -\hat{e}_r \]
\[ \frac{\partial \hat{e}_\phi}{\partial \theta} = 0 \]

\[ \hat{e}_r = \sin \theta \cos \phi \hat{e}_x + \sin \theta \sin \phi \hat{e}_y + \cos \theta \hat{e}_z \]
\[ \hat{e}_\theta = \cos \theta \cos \phi \hat{e}_x + \cos \theta \sin \phi \hat{e}_y - \sin \theta \hat{e}_z \]
\[ \hat{e}_\phi = -\sin \phi \hat{e}_x + \cos \phi \hat{e}_y \]
Use of ‘chain rule’ to get the partial derivatives of the unit vectors using the transformation rules for the unit vectors.

\[
\hat{e}_r = \sin \theta \cos \phi \hat{e}_x + \sin \theta \sin \phi \hat{e}_y + \cos \theta \hat{e}_z
\]

\[
\hat{e}_\theta = \cos \theta \cos \phi \hat{e}_x + \cos \theta \sin \phi \hat{e}_y - \sin \theta \hat{e}_z
\]

\[
\hat{e}_\phi = -\sin \phi \hat{e}_x + \cos \phi \hat{e}_y
\]

For example:

\[
\frac{\partial \hat{e}_\phi}{\partial \phi} = -\cos \phi \hat{e}_x - \sin \phi \hat{e}_y
\]

\[
\frac{\partial \hat{e}_\phi}{\partial \phi} = -\cos \phi \left( \sin \theta \cos \phi \hat{e}_r + \cos \theta \cos \phi \hat{e}_\theta - \sin \phi \hat{e}_\phi \right)
\]

\[\quad - \sin \phi \left( \sin \theta \sin \phi \hat{e}_r + \cos \theta \sin \phi \hat{e}_\theta + \cos \phi \hat{e}_\phi \right)\]

\[
\frac{\partial \hat{e}_\phi}{\partial \phi} = -\sin \theta \hat{e}_r - \cos \theta \hat{e}_\theta
\]

Other partial derivatives can be obtained equally easily, and left for your to do as an exercise!
\[ \frac{\partial \hat{e}_r}{\partial r} = \vec{0} \]
\[ \frac{\partial \hat{e}_r}{\partial \theta} = \hat{e}_\theta \]
\[ \frac{\partial \hat{e}_r}{\partial \phi} = \sin \theta \hat{e}_\phi \]

\[ \frac{\partial \hat{e}_\theta}{\partial r} = \vec{0} \]
\[ \frac{\partial \hat{e}_\theta}{\partial \theta} = -\hat{e}_r \]
\[ \frac{\partial \hat{e}_\theta}{\partial \phi} = \cos \theta \hat{e}_\phi \]

\[ \frac{\partial \hat{e}_\phi}{\partial r} = \vec{0} \]
\[ \frac{\partial \hat{e}_\phi}{\partial \theta} = \vec{0} \]
\[ \frac{\partial \hat{e}_\phi}{\partial \phi} = -\cos \theta \hat{e}_\theta - \sin \theta \hat{e}_r \]

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MOTION IN SPHERICAL POLAR:

VELOCITY AND ACCELERATION

Infinitesimal displacement

Position vector \( \vec{r} = r\hat{e}_r \)

\[
\begin{align*}
\overrightarrow{dr} &= dr\hat{e}_r + r\,d\hat{e}_r \\
\overrightarrow{dr} &= dr\hat{e}_r + r\,d\theta\hat{e}_\theta + r\sin \theta\,d\phi\hat{e}_\phi
\end{align*}
\]

\[
\Rightarrow \overrightarrow{dr} = dr\hat{e}_r + r \frac{\partial \hat{e}_r}{\partial \theta} \delta \theta + r \frac{\partial \hat{e}_r}{\partial \phi} \delta \phi
\]

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Motion in spherical polar:

Velocity and acceleration

\[ d\vec{r} = dr\hat{e}_r + r d\theta \hat{e}_\theta + r \sin \theta d\phi \hat{e}_\phi \]

\[ \vec{v} = \dot{r}\hat{e}_r + r \dot{\theta} \hat{e}_\theta + r \sin \theta \dot{\phi} \hat{e}_\phi \]

\[ \vec{a} = \frac{d\vec{v}}{dt} = (\ddot{r} - r \dot{\theta}^2 - r \sin^2 \theta \dot{\phi}^2) \hat{e}_r + (2\dot{r} \dot{\theta} - r \sin \theta \cos \theta \dot{\phi}^2 + r \ddot{\theta}) \hat{e}_\theta + (2\dot{r} \dot{\phi} \sin \theta + 2r \dot{\phi} \dot{\theta} \cos \theta + r \sin \theta \ddot{\phi}) \hat{e}_\phi \]
General Reference on Vector analysis:


SUPPLEMENTARY OPTIONAL READING:
General Reference on Astronomy:
Carl Sagan: Cosmos

Slightly advanced references:
Arfken: Mathematical Methods for Physicists.
Boas: Mathematical methods in Physical Sciences.

WE WILL TAKE A BREAK...

...... ANY QUESTIONS ?

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Next, Unit 4: Dynamical Symmetry of the Kepler Problem