1. State whether the following statements are ‘TRUE’ or ‘FALSE’ and give reason. The reason should be short, but as rigorous as you can provide.

   a. For a particle of mass \( m \) moves in a region of space where the potential is described by 
   \[
   U(x, y) = -U_0 \exp \left[ -\frac{(x^2 + y^2)}{2L^2} \right],
   \]
   the point \( (x=0, y=0) \) is a ‘saddle point’ (given: \( U_0 \) & \( L \) are positive constants).

   b. If a vector field \( \vec{A} \) is both irrotational \( (\vec{\nabla} \times \vec{A} = \vec{0}) \) and solenoidal \( (\vec{\nabla} \cdot \vec{A} = 0) \), then it must be identically equal to the null vector.

2. A position-dependent force field is given by the expression 
   \[
   \vec{F} = A(x - y)\hat{e}_x + (x + y)\hat{e}_y.
   \]
   It is given that \( |A| = +1 \).

   (a) What is/are the dimension(s) of \( A \)?

   (b) The given force acts on a particle, moving it along a closed path described by the two curves:
   
   \( y = x^2, \) traversed from \((0,0)\) to \((1,1)\),
   
   and
   
   \( y^2 = x \) traversed from \((1,1)\) to \((0,0)\).
   
   Sketch the closed path on the graph sheet in the margin.

   Write your name here: ____________________________

   and submit this graph along with the rest of the answer book when you finish the exam.

   (c) Determine the work \( \int \vec{F} \cdot d\vec{l} \) done by the above force over the closed path described above.

   (d) Without determining the curl of this force \( (i.e. \) without finding \( \vec{\nabla} \times \vec{F} \)), can you tell if the force is irrotational or not? Explain how!
3. A scalar field \( \psi(x, y) \) is given by the expression \( \psi(x, y) = \psi_0 \exp(x^2 + y^2 - 4x - 8y) \),

where \( \psi_0 \) is a constant having suitable dimensions.

(a) Obtain the equipotential curve for \( \psi = \psi_0 \).

(b) Sketch the vector field \( \vec{\nabla} \psi \) at \( \psi = \psi_0 \).

Write your name here: ________________________________

and submit this graph along with the rest of the answer book when you finish the exam.

4. (a) Determine the divergence of the vector point function described by:
\[
\vec{A}(\mathbf{r}) = (r \cos \theta) \hat{e}_r + (r \sin \theta) \hat{e}_\theta + (r \sin \theta \cos \phi) \hat{e}_\phi
\]

(b) Find the flux of the above vector field over a closed surface that encloses a hemisphere of radius \( R \) resting on the xy-plane, with its center at origin and located in the region \( z \geq 0 \).

5. A planet in a remote galaxy rotates rapidly about its own axis. It completes one full rotation in one second. Sketch \( T(\lambda) \) vs \( \lambda \) for this planet, where \( T(\lambda) \) is the time period for the rotation of a Foucault pendulum set in motion on this planet, \( \lambda \) is the latitude; \( -\frac{\pi}{2} \leq \lambda \leq \frac{\pi}{2} \).

Write your name here:

and submit this sheet having the graph along with the rest of the answer book when you finish the exam.