$$X \xrightarrow{\text{outcomes}}$$

$$x_1 \quad x_2 \quad \dots \quad x_M$$

$$p_1 \quad p_2 \quad \dots \quad p_M$$

$$\sum_{i=1}^{M} p_i = 1.$$

$$H(p_1, p_2, \dots p_M)$$

$$f(M) \equiv H\left(\frac{1}{M}, \frac{1}{M} \cdots \frac{1}{M}\right)$$

$f(M)$

If $M > M'$

$f(M) > f(M')$.

$f(M)$ is a monotonically increasing function of M

<u>X, Y</u>

$f(MN) = f(M) + f(N)$

$f(1) = 0$

Grouping Theorem

$X \quad M$

A: r events $\quad x_1 \; x_2 \ldots x_r$

$p_1 \; p_2 \ldots p_r$.

B: $x_{r+1} \; x_{r+2} \ldots x_M$

$p_{r+1} \ldots p_M$.

$$H(p_1, p_2 \ldots p_M) - H\left(\sum_{i=1}^{r} p_i, \sum_{i=r+1}^{M} p_i\right)$$

$$= \sum_{i} p_i \, H\left(\frac{p_1}{\sum_{i=1} p_i}, \frac{p_2}{\cdots}\right) + \sum_{i=r+1}^{M} p_i \, H(\quad)$$

$$A : p_1 = \frac{1}{2}, \quad p_2 = \frac{1}{4}$$

$$B : p_3 = p_4 = \cancel{4}\,\frac{1}{8}$$

$$H\Big(\underbrace{\tfrac{1}{2}, \tfrac{1}{4}}_{A}, \underbrace{\tfrac{1}{8}, \tfrac{1}{8}}\Big) - H\Big(\tfrac{3}{4}, \tfrac{1}{4}\Big)$$

$$= \frac{3}{4} H\Big(\frac{2}{3}, \frac{1}{3}\Big) + \frac{1}{4} H\Big(\frac{1}{2}, \frac{1}{2}\Big)$$

Group A : $p_1 = \frac{1}{2}$, $p_2 = \frac{1}{4}$: $P(A) = \frac{3}{4}$

Group B : $p_3 = p_4 = \frac{1}{8}$: $P(B) = \frac{1}{4}$.

$$H\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right) - H\left(\frac{3}{4}, \frac{1}{4}\right)$$

$$= \frac{3}{4} H\left(\frac{2}{3}, \frac{1}{3}\right) + \frac{1}{4} H\left(\frac{1}{2}, \frac{1}{2}\right)$$