1. Consider the Drude model for semi-classical electron transport (in 3-dimensional bulk). Derive the expression for mobility $\mu$ in terms of the average momentum relaxation time $<\tau_m>$. Estimate $\mu$ for $m^* \sim m_e/10$ and $\tau_m \sim 10^{-13}\text{sec}$.

2. Consider the semi-classical electron transport due to a small electric field along the z-direction. Using the simple Boltzmann Transport Equation (BTE) with single relaxation time approximation, show that the average momentum relaxation time $<\tau_m>$ is determined by

$$<\tau_m> = \frac{\beta m^*}{3\sqrt{\pi}} \int d^3v \frac{\tau_m(v^2)}{\int d^3v f_o(v)},$$

where $\beta = 1/(k_B T)$, $f_o(v)$ is the equilibrium electron distribution given by the Fermi-Dirac distribution, and the integrations are over the infinite velocity space.

For an isotropic case, where $\tau_m(|v|) = \tau_o(\beta \epsilon)^r$, show that

$$<\tau_m> = \frac{4\tau_o}{3\sqrt{\pi}} \left( r + \frac{3}{2} \right) \frac{F_{r+1/2}(x_F)}{F_{1/2}(x_F)},$$

where $x_F = \beta \epsilon_F$ and $F_j(x)$ are the so-called Fermi integrals defined as

$$F_j(x) = \frac{1}{j!} \int_0^\infty \frac{y^j}{e^{y-x} + 1} dy.$$

Obtain the expressions for $<\tau_m>$ in the two opposite limits, $T \to 0$ and $T \to \infty$.

3. Using the linearized BTE for a quasi 2-DEG system, and assuming that the scattering rate is elastic, i.e., $\epsilon(k') = \epsilon(k) = \epsilon$ (say), show that the relaxation time for nth sub-band is given by

$$\frac{1}{\tau_n(\epsilon)} = \sum_m \sum_{k'} S_{nn}(k', k') \left[ 1 - \frac{\tau_m(\epsilon)}{\tau_n(\epsilon) \cos \theta_{kk'}} \right],$$

where all quantities have their usual meaning.

Finally assuming $S_{nm} \approx 0$ for $n \neq m$, and $\tau_n = \tau_no(\beta \epsilon)^r$ obtain the general expression for $<\tau_n>$, and its form in the two extreme temperature limits.

4. Using Fermi’s Golden rule for the scattering rate for scattering from charged impurities, show under appropriate assumptions, that for a 2-DEG system $\tau_n(E)$ is proportional to $E$.

5. Using Fermi’s Golden rule for the scattering rate for scattering from roughness at the interface, under appropriate assumptions for a 2-DEG system obtain an expression for $\tau_n(E)$.

6. Using Fermi’s Golden rule for the scattering rate for scattering from charged impurities, under appropriate assumptions for a quantum wire system obtain an expression for the scattering rate $S_{nn}$. 
References:

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