Multi-Channel:
It may happen that even at low temperature, there are \( N \) number of 1d sub-bands (modes) which are populated at the Fermi energy, all of which contribute to the current. Alternatively, at finite temperature, one could view the situation as a multi-channel case, where each channel represents a particular narrow range of energy. On the source side on the left, the wave function in the ideal lead on the left having \( N \) sub-bands populated, may be written as

\[
\psi^L(\vec{r}, z) = \sum_{n=1}^{N} \left[ A_n^L e^{i k_{n,z} z} + B_n^L e^{-i k_{n,z} z} \right] \phi_n(\vec{r}_t),
\]

where \( \vec{r}_t \) is the position vector in the transverse direction, \( n \) labels the transverse mode, \( k_{n,z} \) is the wave vector and \( r_{nt} \) is the reflection coefficient of the wave on the left. Similarly, on the drain side on the right having \( N' \) sub-bands populated, the wave in the ideal lead on the right, may be written as

\[
\psi^R(\vec{r}, z) = \sum_{n'=1}^{N'} \left[ A_n^R e^{i k_{n',z} z} + B_n^R e^{-i k_{n',z} z} \right] \phi_{n'}(\vec{r}_t).
\]

Thus the left lead involves \( 2N \) coefficients \( (A_{n1}^L, B_{n1}^L, n = 1, \ldots, N) \) and the right lead involves \( 2N' \) coefficients \( (A_{n1}^R, B_{n1}^R, n' = 1, \ldots, N') \), and scattering (or tunneling) processes connect them (via a scattering or transfer matrix). For simplicity, we assume that the two ideal leads are identical so that \( N = N' \). The incoming plane wave in the mode \( \phi_n \) in the left lead has certain probability \( T_{n'n} = |t_{n'n}|^2 \) to be transmitted to the mode \( \phi_{n'} \) in the right lead, and probability \( R_{n'n} = |r_{n'n}|^2 \) of being reflected back. The transmission and reflection amplitudes \( t_{n'n} \) and \( r_{n'n} \) are the elements of the \( 2N \times 2N' \) scattering matrix.

Now suppose that the carriers are fed equally into all these modes in the leads from the reservoirs (electrodes) up to the Fermi energy \( \mu_1 \) on the left, and \( \mu_1 \) on the right. For a particular mode \( \phi_{n'} \) on the left, the current into the channel \( \phi_n \) on the right is then

\[
I_{n'n'} = \frac{2e}{h} T_{n'n} (\mu_1 - \mu_2).
\]

Since each channel is fed equally, the total current into the \( n^{th} \) mode is

\[
I_n = \frac{2e}{h} (\mu_1 - \mu_2) \sum_{n'=1}^{N} T_{n'n} = \frac{2e}{h} (\mu_1 - \mu_2) T_n,
\]

denoting \( \sum_{n'=1}^{N} T_{n'n} \) as \( T_n \).

Since all channels are assumed independent, the total current is

\[
I_{Total} = \frac{2e}{h} (\mu_1 - \mu_2) \sum_{n=1}^{N} T_n.
\]

Similarly, the current into the channel \( \phi_n \) on the left side is

\[
I_n = \frac{2e}{h} (\mu_1 - \mu_2) [1 - R_n],
\]

where \( R_n = \sum_{n'=1}^{N} R_{n'n'} \), and the corresponding total current is

\[
I_{Total} = \frac{2e}{h} (\mu_1 - \mu_2) \sum_{n=1}^{N} [1 - R_n].
\]

Comparison of the expressions for \( I_{Total} \) gives the current continuity equation between all channels as

\[
\sum_{n=1}^{N} T_n = \sum_{n=1}^{N} [1 - R_n].
\]

As in the single channel case, the ideal leads are labeled by \( \mu_A \) (on left) and \( \mu_B \) (on right) to account
for the self consistent pile up of charges on either side of the structure. To relate $\mu_A$ and $\mu_B$ to $\mu_1$ and $\mu_2$, note that for the right side, the total number of occupied states in the range $\mu_B$ to $\mu_1$ on the right is

$$n_{\text{occ}}^r = \sum_{n=1}^{N} T_n D_n(E) (\mu_1 - \mu_B), \tag{30}$$

where, $D_n(E) = 1/(\pi \hbar v_n)$ is the density of states with positive velocity $v_n$ in the $n$th mode. Similarly the total number of unoccupied states in the range $\mu_2$ to $\mu_B$ on the right is

$$n_{\text{unocc}}^r = 2 \sum_{n=1}^{N} D_n(E) (\mu_B - \mu_2) - \sum_{n=1}^{N} T_n D_n(E) (\mu_B - \mu_2), \tag{31}$$

where on the right hand side the first term corresponds to the total number of states (both for positive and opposite velocities) and the second term corresponds to the number of states to which the carriers are injected from the left.

For consistency these two must be equal, and therefore, equating the two, one gets

$$\sum_{n=1}^{N} T_n D_n(E) (\mu_1 - \mu_B) = \sum_{n=1}^{N} [2 - T_n] D_n(E) (\mu_B - \mu_2),$$

so that, using $D_n = 1/(\pi \hbar v_n)$, one gets

$$\mu_B = \mu_2 + \frac{(\mu_1 - \mu_2)}{2} \frac{\sum_{n=1}^{N} T_n v_n^{-1}}{\sum_{n=1}^{N} v_n^{-1}}. \tag{32}$$

Similarly considering states on the left, one gets

$$\sum_{n=1}^{N} [1 + R_n] D_n(E) (\mu_1 - \mu_A) = \sum_{n=1}^{N} [1 - R_n] D_n(E) (\mu_A - \mu_2),$$

so that,

$$\mu_A = \frac{\mu_1 + \mu_2}{2} + \frac{(\mu_1 - \mu_2)}{2} \frac{\sum_{n=1}^{N} R_n v_n^{-1}}{\sum_{n=1}^{N} v_n^{-1}}. \tag{33}$$

Therefore,

$$eV = \mu_A - \mu_B = \frac{(\mu_1 - \mu_2)}{2} \frac{\sum_{n=1}^{N} [R_n - T_n] v_n^{-1}}{\sum_{n=1}^{N} v_n^{-1}}. \tag{34}$$

Hence, when the current and voltage are measured independently as done in a four terminal measurement, the conductance is

$$G_4 = \frac{I_{\text{Total}}}{V} = \frac{2e^2}{h} \frac{2 \sum_{n=1}^{N} T_n}{1 + \frac{\sum_{n=1}^{N} [R_n - T_n] v_n^{-1}}{\sum_{n=1}^{N} v_n^{-1}}}. \tag{35}$$

If one performs a two terminal measurement, such that one measures the potential drop across both the structure and the leads, then the conductance is

$$G_2 = \frac{2e^2}{h} \sum_{n=1}^{N} T_n. \tag{36}$$

If one considered an unequal number of channels on either side of the scatterer (e.g., if the width of the ideal 1D conducting leads were different on the two sides, so that the number of channels are $N$ on left and $N'$ on right), and if the velocities are also different, then one can generalize the result above (for details, see, Buttiker et.al., Phys Rev B31, (1985) 6207) to

$$G_4 = \frac{2e^2}{h} \frac{2 \sum_{n=1}^{N'} T_n}{1 + \frac{\sum_{n=1}^{N} R_n v_n^{-1}}{\sum_{n=1}^{N} v_n^{-1}} - \frac{\sum_{n=1}^{N'} T_n v_{r,n}^{-1}}{\sum_{n=1}^{N'} v_{r,n}^{-1}}}, \tag{37}$$

which reduces to the symmetric result if the velocities are same on both sides. If the reflection
coefficients are nearly unity, and the transmission coefficients small, then the four terminal conductance $G_4$ reduces to the two terminal result $G_2$.

Reference

- Ferry D. K and Goodwick S M