Consider a sample of a small quantum wire structure with only one sub-band (channel) occupied. Ideal (i.e., resistance free or without scattering) conducting leads (A and B) connect the quantum wire sample which act as scattering region, to reservoirs (electrodes) on the left (Source, labeled 1) and right (drain labeled 2) having quasi Fermi energies $\mu_1$ and $\mu_2$ respectively, corresponding to the electron densities there (see Fig. 6.3). The reservoirs randomize the phase of the injected (from source) and absorbed (at drain) electrons through inelastic processes so that there is no phase relationship between the particles at the two ends.

For such an ideal situation, the current from left (source) to right (drain) is

$$I = 2e \left[ \int_0^\infty \frac{dk}{2\pi} v(k) f_1(k) T_{l\rightarrow r}(E_k) - \int_0^\infty \frac{dk'}{2\pi} v(k') f_2(k') T_{r\rightarrow l}(E'_{k'}) \right],$$

(13)

where $f_1$ and $f_2$ are the reservoir distribution functions characterized by $\mu_1$ and $\mu_2$ respectively, and $T$'s are the transmission coefficients due to scattering by the quantum wire sample, which is regarded as a barrier.

At very low temperatures, electrons are injected with energies up to $\mu_1$ in $Res_1$ and $\mu_2$ in $Res_2$. Converting the $k$-integrals to energy integrals, then at very low temperatures,

$$I = \frac{e}{\pi} \int_0^{\mu_1} \frac{dE}{dE} \frac{dk}{dE} v(k) f_1(k) T(E) - \int_0^{\mu_2} \frac{dE}{dE} \frac{dk'}{dE} v(k') f_2(k') T(E) \Bigg|_{\mu_2} \Bigg|_{\mu_1} = \frac{e}{\pi} \int_{\mu_2}^{\mu_1} dE T(E),$$

(14)

here we have used $T_{l\rightarrow r} = T_{r\rightarrow l} = T(E)$ and $v(k) = \hbar^{-1} (dE/dk)$ in 1D.

If we now assume that the applied voltage is small (i.e., in the linear response regime), so that the energy dependence of $T(E)$ in the range $\mu_2$ to $\mu_1$ is negligible, then, we can write

$$I = \frac{2e}{\hbar} T(\bar{\mu})(\mu_1 - \mu_2),$$

(15)

where $\bar{\mu} = (\mu_1 + \mu_2)/2$.

Now, as a result of transmission and reflection about a barrier, there is a reduction in carrier density on the left of the barrier (i.e., sample), and a pile up of charges on the right side, when a forward bias is
applied. If this charge rearrangement is approximated by an average density, then the actual voltage drop across the sample is given by
\[ eV = \mu_A - \mu_B < \mu_1 - \mu_2, \]
so that the difference between \((\mu_A - \mu_B)\) and \((\mu_1 - \mu_2)\) will appear as the potential drop at the contacts.

Now the 1D density in the ideal lead on the left hand side is
\[ n_A = 2 \int_{-\infty}^{\infty} \frac{dk}{2\pi} f_A(E_k) = \frac{1}{\pi} \left[ \int_0^\infty dk \left[ 1 + R(E_k) \right] f_1(E_k) + \int_0^\infty dk \frac{T(E_k)}{2} f_2(E_k) \right], \]
\[ = \frac{1}{\pi} \int_0^\infty dk \left[ 2 - T(E_k) \right] f_1(E_k) + T(E_k) f_2(E_k), \]
using \( R = 1 - T \). Similarly,
\[ n_B = 2 \int_{-\infty}^{\infty} \frac{dk}{2\pi} f_B(E_k) = \frac{1}{\pi} \int_0^\infty dk \left[ 2 - T(E_k) \right] f_2(E_k) + T(E_k) f_1(E_k). \]

Now taking low temperature limit,
\[ n_A - n_B = 2 \int_{\mu_B}^{\mu_A} dE \left( \frac{dk}{dE} \right) = 2 \int_{\mu_2}^{\mu_1} dE \left( \frac{dk}{dE} \right) \left[ 1 - T(E_k) \right]. \]

Now assume that the differences in the fermi energies are sufficiently small so that the energy dependence of \( T \) and \( dE/dk \) may be neglected, and the integrals readily performed to get
\[ \mu_A - \mu_B = [1 - T] (\mu_1 - \mu_2), \]
where \( T \equiv T(\tilde{\mu}) \); note that it is tacitly assumed that \( \tilde{\mu} = (\mu_1 + \mu_2)/2 \approx (\mu_A + \mu_B)/2 \).

Using above in the expression for \( I \) then gives
\[ I = \frac{2e}{h} \frac{T}{1 - T} (\mu_A - \mu_B) = \frac{2e^2}{h} \frac{T}{1 - T} V. \]

The conductance is then given by
\[ G_4 = \frac{I}{V} = \frac{2e^2}{h} \frac{T}{1 - T}, \] (Four-probe formula),
which is the Single channel Landauer formula.

Note the appearance of the fundamental conductance \( 2e^2/h = 7.748 \times 10^{-5} \text{ mhos} \) (corresponding to a resistance of 12,907 \( \Omega \)). Note also that the above formula implicitly assumed that one has applied voltage through a pair of contacts and measured the voltage difference in the ideal leads non-invasively through a separate pair of contacts (as done in a Four-terminal measurements). If on the other hand one performed a Two-terminal measurement, i.e., the voltage and current measurement through the same set of leads, then one would use \( eV = \mu_1 - \mu_2 \) and obtain
\[ G_2 = \frac{I}{V} = \frac{2e^2}{h} T, \] (Two-probe formula),
without the factor \( (1 - T) \) in the denominator. The reason is that now, in addition to the potential drop across the structure, one is measuring a contact potential drop across the ideal leads due to the self-consistent charge build up characterized by a contact resistance \( R_{\text{contact}} = h/(2e^2) \). If \( T \) is small, then the series resistance of the barrier would dominate, and the two-terminal and four-terminal measurements would give indistinguishable results.