Resonant tunnel diode (RTD):
A resonant tunnel diode is typically made of heterostructures as shown below.

![Diagram of a resonant tunnel diode](image)

**Fig 5.1 : Typical tunneling in double barrier with and without bias**

The extreme left is GaAs with donor concentration \(N_{d1}\). Next is a layer of \(\text{Al}_{x}\text{Ga}_{1-x}\text{As}\) of width \(w_1\) and Al-mole fraction \(x\). Next to the right is again GaAs with donor concentration \(N_{d2}\) and width \(w_2\), followed by the layer of \(\text{Al}_{x}\text{Ga}_{1-x}\text{As}\) with same \(x\) as before and width \(w_3\) same as \(w_1\). The last layer is the GaAs with donor concentration \(N_{d3}\). Typically, \(N_{d1} = N_{d3} = 10^{18}\text{cm}^{-3}\) and \(N_{d2} = 10^{17}\text{cm}^{-3}\), \(w_1 = w_2 = w_3 \approx 50\text{Å} \approx 5\text{nm}\). This is a structure where tunneling takes place across two barriers. There may be one or two or more barriers depending on the number of AlGaAs layers used.

The typical I-V characteristic of RTD is as shown in Fig 5.2.

![Graph of typical I-V curves for RTD](image)

**Fig 5.2 : Typical I-V Curves for RTD**

The main feature is the peak at some voltage when the RTD is at low temperature. Thin peak is washed out at high temperature. The aim now is to understand the origin of the low temperature I-V characteristic of the RTD.

**Single rectangular symmetric barrier:**
The simplest is the single rectangular barrier as shown in Fig 5.3.

![Graph of single rectangular symmetric barrier](image)

**Fig 5.3 : Single rectangular symmetric barrier**

For a theoretical treatment, the simplest model is the single band effective mass model \(V_0 = \Delta E_C\) at
the $\Gamma - \text{point}$ in the two materials for incident electron energy lower than the band energy at X-point in both the materials, where X-point is the B-zone boundary point along 0 0 1, close to which the second band minimum occurs. If this condition is not satisfied, i.e., electron energy is more than the second band minimum, then one band model is no longer valid. Then the basic equation of concern is the effective mass equation:

$$\left[ -\frac{\hbar^2}{2} \frac{\partial}{\partial t} \frac{1}{m^*(z)} \frac{\partial}{\partial z} + V_{\text{eff}}(z) \right] \varphi(z) = E \varphi(z)$$

If the space-charge effects are negligible, then $V_{\text{eff}}(z)$ is simply that due to band offset $\Delta E_C$. Note that $E$ is the energy of incoming electron (not the stationary eigenstates for the system).

![Fig 5.4: Band diagram for single barrier](image)

Now choose the coordinate system as shown in Fig 4.4, with $z < |a|$ marked Region II and region $z < -a$ marked Region I and region $z > a$ marked region III.

Then,

$$\varphi(z) = A_1 e^{ikz} + B_1 e^{-ikz} \quad \text{in I}$$

$$= A_2 e^{ikz} + B_2 e^{-ikz} \quad \text{in II}$$

$$= A_3 e^{ikz} + B_3 e^{-ikz} \quad \text{in III}$$

where $k = \sqrt{2m^*E/\hbar^2}$, $\gamma = \sqrt{2m^*(V_0 - E)/\hbar^2}$ \left( \gamma/k = \sqrt{V_0/E - 1} \right)$

Note that the coefficient A's are associated with (incoming) waves propagating along (+)ve x-axis, while B's are associated with propagation along (-)ve x-axis. For simplicity, it is assumed that $m_1^* = m_2^* = m_3^* = m^*$

The matching conditions are (at $z = -a$)

$$\varphi_1(-a-\epsilon) = \varphi_1(-a+\epsilon), \quad \epsilon > 0 \text{ is a positive infinitesimal}$$

$$\frac{1}{m_1^*} \left. \frac{\partial \varphi_1}{\partial z} \right|_{z=-a-\epsilon} = \frac{1}{m_2^*} \left. \frac{\partial \varphi_2}{\partial z} \right|_{z=-a+\epsilon}$$

or

$$\left. \frac{\partial \varphi_1}{\partial z} \right|_{z=-a-\epsilon} = \left. \frac{\partial \varphi_2}{\partial z} \right|_{z=-a+\epsilon}$$

(since $m_1^* = m_2^* = m^*$ as assumed)

Then $A_1 e^{-ikz} + B_1 e^{ikz} = A_2 e^{-\gamma z} + B_2 e^{\gamma z}$

$$ik \left[ A_1 e^{-ikz} + B_1 e^{ikz} \right] = \gamma \left[ A_2 e^{-\gamma z} - B_2 e^{\gamma z} \right]$$

or

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} \frac{(ik + \gamma)}{2ik} e^{(ik - \gamma)z} & \frac{(ik - \gamma)}{2ik} e^{(ik + \gamma)z} \\ \frac{(ik - \gamma)}{2ik} e^{-ikz} & \frac{(ik + \gamma)}{2ik} e^{-ikz} \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \end{bmatrix}$$

Similarly, the matching condition at $z = a$ gives
or \[
\begin{bmatrix}
A_2 \\
B_2
\end{bmatrix} = \begin{bmatrix}
\frac{ik + \gamma}{2\gamma} e^{(\bar{a} - \gamma)\alpha} & -\frac{ik - \gamma}{2\gamma} e^{-(\bar{a} + \gamma)\alpha} \\
-\frac{ik - \gamma}{2\gamma} e^{(\bar{a} + \gamma)\alpha} & \frac{ik + \gamma}{2\gamma} e^{-(\bar{a} - \gamma)\alpha}
\end{bmatrix} \begin{bmatrix}
A_3 \\
B_3
\end{bmatrix}
\]

Combining the two results, one may write

\[
\begin{bmatrix}
A_1 \\
B_1
\end{bmatrix} = \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix} \begin{bmatrix}
A_3 \\
B_3
\end{bmatrix}
\]

where

\[
M_{11} = \left(\frac{ik + \gamma}{2ik}\right)\left(\frac{ik + \gamma}{2\gamma}\right) e^{2(\bar{a} - \gamma)\alpha} - \left(\frac{ik - \gamma}{2k}\right)\left(\frac{ik - \gamma}{2\gamma}\right) e^{2(\bar{a} + \gamma)\alpha}
\]

\[
= \left[\cosh 2\gamma \alpha - \frac{i}{2} \left(\frac{k^2 - \gamma^2}{k\gamma}\right) \sinh 2\gamma \alpha\right] e^{2\kappa a}
\]

\[
M_{21} = \left(\frac{ik - \gamma}{2k}\right)\left(\frac{ik + \gamma}{2\gamma}\right) e^{-2\gamma \alpha} - \left(\frac{ik + \gamma}{2ik}\right)\left(\frac{ik - \gamma}{2\gamma}\right) e^{2\gamma \alpha}
\]

\[
= -\frac{i}{2} \frac{k^2 + \gamma^2}{k\gamma} \sinh 2\gamma \alpha
\]

\[
M_{22} = M_{11}^*, \quad M_{21} = M_{21}^*
\]

Note that \(\det M = 1\), but \(M\) (called transmission or transfer matrix) is not a unitary matrix, as its diagonal elements are complex.