The density of states for a quasi 1D system of rectangular quantum wave guide is

\[ D(E) = \frac{1}{L_z} \sum_{n,m} \delta \left( E - E_{n,m} (k_z) \right) \]

\[ \text{no. of available electron states per unit length} \]

for \( L_z \) large,

\[ 2m_r \sum_{n,m} \int \frac{dk_z}{2\pi} \delta \left( E - E_{n,m} - \frac{\hbar^2 k_z^2}{2m_r} \right) \]

\[ = \frac{n_e}{\pi} \left( \frac{2m_r^*}{\hbar^2} \right)^{1/2} \sum_{n,m} \frac{\Theta(E - E_{n,m})}{\sqrt{E - E_{n,m}}} \]

**Fig 2.13: Schematic density of states for a quasi 1D system**

The density of states now have the structure as shown in Fig 2.13, with divergences at sub band minima.

**Special Case:** Consider the special case where

\[ L_x = L, \ L_y = L_z = w \ll L \]

Then

\[ E_{n,m} = \frac{\hbar^2 \pi^2}{2m^*_w} \left( n^2 + m^2 \right), \ n = 1, 2, 3 \ldots. \]

Clearly some of the levels are degenerate (apart from spin degeneracy) because of the special choice of \( L_y = L_z \). The levels are listed below:

Let \( E_0 = \frac{\hbar^2 \pi^2}{2m^*_w} \). Then

\[ E_{n,m} = E_0 \left( n^2 + m^2 \right) \]
Lowest level: \((1,1)\) : \(E_1 = E_0 (1+1) = 2 E_0\), degeneracy: 1

Next level: \((1,2)\) \(E_2 = E_0 (1+4) = 5 E_0\), degeneracy: 2

\((2,1)\)

Next level: \((2,2)\) : \(E_3 = E_0 (4+4) = 8 E_0\), degeneracy: 1

Next level: \((1,3)\)

\((3,1)\) : \(E_4 = E_0 (1+9) = 10 E_0\), degeneracy: 2

Next level: \((2,3)\)

\((3,2)\) : \(E_5 = E_0 (4+9) = 13 E_0\), degeneracy: 2

Next level: \((1,4)\)

\((4,1)\) : \(E_6 = E_0 (1+16) = 17 E_0\), degeneracy: 2

Next level: \((2,4)\)

\((4,2)\) : \(E_7 = E_0 (4+16) = 20 E_0\), degeneracy: 2

Note that in atomic units (a.u.) \(\hbar = 1\), \(m_e = 1/2\) so that length is unit of \(a_B\) (Bohr radius = 0.053 nm) and energy is in unit of Rydberg (= 13.6 eV); in a.u. one then has; \(E_{n,m} = (n^2 + m^2)E_0\) with \(E_0(m_e/m_e^*)n^2(w/a_B)^{-2} \approx 10(w/a_B)^{-2}\) Ryd = 136(w/a_B)^{-2} eV for \(m_e \approx m^*\). Thus for \(w \approx 15\) nm, \(E_0 \approx 1.5\) meV.

For GaAs quantum wire, \(E_0 \approx 0.0125\) eV = 12.5 meV (a typical value).

Note that the divergences in the density of states may lead to interesting effects in the transport properties and collective excitations of such systems. However in reality, these divergences are usually broadened to finite peaks due to disorder and no strong effects have so far been observed experimentally.