Semiconductor Statistics and Density of states:

Density of states for conduction band is

\[ D_e(E) \, dE = \frac{1}{2\pi^2} \left( \frac{2m_e^*}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2} \, dE \quad E \geq E_c \quad \{E_c = \text{conduction band edge}\} \]

For direct gap semiconductors, \(m_e^*\) is the effective mass, \(m_e^*\) appearing in the \(E(k)\)-relation of the form

\[ E(k) = E_c + \frac{\hbar^2 k^2}{2m_e^*}. \]

For a material like Si with different effective masses and six different conduction-band valleys, the conduction band density of states mass to be denoted by \(m_{dos}^*\) should replace above in \(D_e\) where \(m_{dos}^*\) is defined by

\[ m_{dos}^* = \left( m_e^* \eta_v \eta_v^* \right)^{1/3}, \quad \eta_v = \text{valley degeneracy} = 6 \text{ here} \]

If the valence band \(E(k)\)-relation is of the form

\[ E(k) = E_v - \frac{\hbar^2 k^2}{2m_h^*} \quad (E_v \equiv \text{top of valence band}), \]

then,

\[ D_h(E) \, dE = \frac{1}{2\pi^2} \left( \frac{2m_h^*}{\hbar^2} \right)^{3/2} (E_v - E)^{1/2} \, dE \quad E \leq E_v \]

Note that the valence band density of states is zero for \(E_c > E > E_v\), i.e., \(E\) in the band gap region.

When the valence band can be represented by heavy-hole and light-hole bands, then summing the density of states for these two bands, one gets

\[ D_h(E) \, dE = \frac{1}{2\pi^2} \left( \frac{2m_{h}^*}{\hbar^2} \right)^{3/2} (E_v - E)^{1/2} \left[ \left( m_{hh}^* \right)^{3/2} + \left( m_{lh}^* \right)^{3/2} \right] \, dE \]

\[ \equiv \frac{1}{2\pi^2} \left( \frac{2m_{dos}^*}{\hbar^2} \right)^{3/2} (E_v - E)^{1/2} \, dE \]

where \(m_{dos}^* \equiv \left( m_{hh}^* \right)^{3/2} + \left( m_{lh}^* \right)^{3/2}\)

For Si and GaAs, one finds

\[ m_{dos}^* \quad (\text{conduction band}) = 1.08 \, m_e \quad \text{for Si}, \quad 0.067 \, m_e \quad \text{for GaAs} \]

\[ m_{dos}^* \quad (\text{valence band}) = 0.55 \, m_e \quad \text{for Si}, \quad 0.47 \, m_e \quad \text{for GaAs} \]
Carrier Densities:

The concentration of electrons in the conduction band is

\[
n = \int_{E_c}^{\infty} D_e(E) f_e(E) \, dE, \quad f_e(E) = \frac{1}{e^{\beta(E - \mu)} + 1} \quad (\text{Fermi-function})
\]

\[
= \frac{1}{2\pi^2} \left( \frac{2m_e^*}{\hbar^2} \right)^{3/2} \int_{E_c}^{\infty} \frac{(E - E_c)^{1/2}}{e^{\beta(E - \mu)} + 1} \, dE \quad \text{\left(no./cm^3\right)}
\]

where \(\mu\) = chemical potential and (loosely speaking, the Fermi energy) and \(\beta = 1/(k_B T)\).

If the chemical potential \(\mu\) is far from band edge, then the unity in the denominator of the Fermi-function may be neglected. This approximation, called Boltzmann Approximation is valid for small \(n\) \((\leq 10^{28} \text{ m}^{-3} \text{ for most semiconductors})\) and is usually applicable to intrinsic concentrations. Then one gets

\[
n = \frac{1}{2\pi^2} \left( \frac{2m_e^*}{\hbar^2} \right)^{3/2} e^{\beta\mu} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{-\beta E} \, dE
\]

\[
= 2 \left( \frac{m_e^* k_B T}{2\pi\hbar^2} \right)^{3/2} e^{\beta(\mu - \xi_c)} \equiv N_c e^{\beta(\mu - \xi_c)}
\]

where \(N_c = 2 \left( \frac{m_e^* k_B T}{2\pi\hbar^2} \right)^{3/2}\), called effective density of states at the conduction band edge, with \(m_e^* = m_{dx}^*\) for electrons.

This gives \(\mu - E_c = k_B T \ln \frac{n}{N_c}\), in Boltzmann approx.

If \(f(E)\) approaches unity in the range of integration, then the above approximation requires modification, and an approximation suggested by Joyce and Dixon is useful. The Joyce-Dixon approximation is given by

\[
\mu = E_c + k_B T \left[ \ln \frac{n}{N_c} + \frac{1}{\sqrt{8N_c}} \right].
\]