Non-Debye relaxation models:
There are several materials (e.g., polymers, glass, etc.), where a single relaxation time does not work. One therefore generalizes the Debye relaxation function \( \tilde{f}(i) \) to the so-called non-Debye relaxation function as

\[
\tilde{f}(\tilde{t}) = A \int_0^\infty \tilde{F}(\tau) e^{-i\tilde{t} \tau} d\tau,
\]

where the distribution function for the relaxation time is normalized, i.e., \( \int_0^\infty \tilde{F}(\tau) d\tau = 1 \). This gives

\[
\varepsilon(\omega) = \varepsilon_{\infty} + A \int_0^\infty \frac{\tilde{F}(\tau)}{1 - i\omega \tau} d\tau.
\]

One sometimes introduces a reduced relaxation time \( \tilde{\tau} = \tau / \tau_D \), and a corresponding distribution

\[
G(\tilde{\tau}) = \tau_D \tilde{F}(\tau_D \tilde{\tau}),
\]

so that

\[
\varepsilon(\omega) = \varepsilon_{\infty} + A \int_0^\infty \frac{G(\tilde{\tau})}{1 - i\omega \tau_D} d\tilde{\tau},
\]

where \( G \) is also normalized, i.e., \( \int_0^\infty G(\tilde{\tau}) d\tilde{\tau} = 1 \). There has been several attempts to derive

\[
\tilde{f}(\omega) = A \int_0^\infty \frac{\tilde{F}(\tau)}{1 - i\omega \tau} d\tau.
\]

from first principle, and is subject of Complex Systems, in particular for biological systems, but without much success. On the other hand, one has empirical forms available as

\[
\tilde{f}(\omega) = \frac{A}{[1 - (-i\omega \tau_D)^\mu]^\nu}.
\]

This is known as Havriliak-Negami formula where \( \mu \) and \( \nu \) are empirical parameters, depending upon the nature of the experimentally observed response; usually one finds \( 0 < \mu \leq 1 \) and \( \mu \nu \leq 1 \). When \( \mu = 1 = \nu \), one gets the classical Debye relaxation. When \( \mu = 1 \) and \( \nu \neq 1 \), then the formula is known as Davidson-Cole formula. When \( \mu \neq 1 \) and \( \nu = 1 \), then the formula is known as Cole-Cole formula.

A further empirical generalization of above is

\[
\tilde{f}(\omega) = \sum_j \frac{A_j}{[1 - (-i\omega \tau_j)^\mu]^\nu},
\]

where the sum over \( j \) represents different dielectric relaxation processes presumably arising from different components in the material.

Note that, when DC conduction is taken into account, then one writes,

\[
\varepsilon(\omega) = \varepsilon_{\infty} + \tilde{f}(\omega) + i \frac{\tilde{\sigma}(\omega)}{\omega},
\]

where \( \tilde{\sigma}(\omega) \) is the complex frequency dependent conductivity usually given by

\[
\tilde{\sigma}(\omega) = \frac{\sigma_\infty}{[1 - i\omega \tau_D]},
\]
where \( \sigma_o = ne^2 \tau/(m \epsilon_o) \) is the dc conductivity.

It can be shown that when

\[
G(\omega) = \Theta(1 - \bar{\omega}) \frac{\bar{\omega}^{-\nu} \sin(\pi \nu)/\pi}{1 + \bar{\omega}^{2\mu} + 2\bar{\omega}^\mu \cos(\pi \mu)},
\]

one gets the Davidson-Cole formula; when

\[
G(\omega) = \frac{\bar{\omega}^{-\mu} \sin(\pi \mu)/\pi}{[1 + \bar{\omega}^{2\mu} + 2\bar{\omega}^\mu \cos(\pi \mu)]},
\]

one gets the Cole-Cole formula; when

\[
G(\omega) = \frac{\bar{\omega}^{-\nu} \sin(\pi \nu)/\pi}{[1 + \bar{\omega}^{2\mu} + 2\bar{\omega}^\mu \cos(\pi \mu)]^{\nu/2}},
\]

where \( \tan \Theta = \sin(\pi \mu)/[\bar{\omega}^\mu + \cos(\pi \mu)] \), one gets the Havriliak-Negami formula.

How such distributions can arise from physical processes in biocells, is far from being understood. For biological systems, it is generally found that the Cole-Cole model is a better fit to experimental data, and for most biological tissues one need not go as far as Havriliak-Negami formula for a good fit.

The Non-Debye relaxation formulas of above empirical type have been applied to various diverse systems such as, polymers, brine saturated rocks, colloidal suspensions, liquid crystals, liquid-liquid mixture of dielectrics, confined polymers where guest molecules are trapped between regular matrices of polymers or on substrates, charge-relaxation phenomena in mixtures composed of ceramics, metal oxides, and ionic conductors.

To summarize, we have briefly outlined various dielectric functions which turn out to be extremely useful for parametrization and interpretation of experimental data. It turns out they can also be used fruitfully as input to the theory of optical absorptions and scattering by nano-particles, as will be seen later.

Reference

- C F Bohren and D R Huffman, Absorption and Scattering of Light by Small Particles, Wiley Interscience, Paperback, 1998, Chapter 9