5. Attenuation of homogeneous plane wave:

For isotropic homogeneous medium described by local responses, usually one writes $N = n + ik'$ and then the form of a homogeneous plane wave in such a medium is

$$\vec{E}(\vec{x}, t) = \vec{E}_0 e^{-\frac{2\pi k}{\lambda_0} z} e^{i\frac{2\pi n}{\lambda_0} z - \omega t}, \quad z = \hat{q} \cdot \vec{x}.$$ 

Clearly the phase velocity is $v = c/n$, along $\hat{q}$. The entities $n$ and $k$ are called optical constants (although they are not really constants, depending on $\omega$) which are actually related via the so called Kramer-Kronig's relations, described by

$$n(\omega) - 1 = \frac{2}{\pi} P \int_0^\infty \frac{k(\omega') \omega'}{\omega^2 - \omega'^2} d\omega', \quad k(\omega) = -\frac{2\omega}{\pi} P \int_0^\infty \frac{[n(\omega') - 1]}{\omega^2 - \omega'^2} d\omega'.$$

The time averaged Poynting vector for time harmonic fields is given by (consult Jackson)

$$< \vec{S} > = \frac{1}{2} \text{Re} \left[ \vec{E} \times \vec{H}^* \right].$$

which for plane waves gives

$$< \vec{S} > = \text{Re} \left[ \frac{\vec{E} \times (\vec{q}^* \times \vec{H}^*)}{2\omega \mu^*} \right] = \text{Re} \left[ \frac{\vec{q}^* (\vec{E} \cdot \vec{E}^*) - \vec{E}^* (\vec{q}^* \cdot \vec{E})}{2\omega \mu^*} \right].$$

For a homogeneous wave, $\vec{q} \cdot \vec{E} = 0$, which also implies $\vec{q}^* \cdot \vec{E} = 0$, and therefore

$$< \vec{S} > = \frac{1}{2} \text{Re} \left[ \frac{\vec{E}_0^2}{\mu} e^{-\frac{4\pi k}{\lambda_0} z} \right].$$

The magnitude of $< \vec{S} >$ is called irradiance $I$. As a homogeneous plane wave traverses the medium, the irradiance is exponentially attenuated as

$$I = I_0 e^{-\alpha z}, \quad z = \hat{q} \cdot \vec{x}.$$ 

where $\alpha = 4\pi k/\lambda_0$ is called the absorption coefficient. Measurements on attenuation of irradiance provide information about the imaginary part of the complex refractive index.

6. Polarization of homogeneous plane wave:

As seen earlier, for a medium with real $N$, one introduces mutually orthogonal unit vectors ($\hat{e}_1, \hat{e}_2, \hat{q}$), to represent two general linearly polarized plane electromagnetic waves propagating along $\hat{q}$ direction as

$$\vec{E}_1(\vec{x}, t) = \hat{e}_1 E_1 e^{i(q \hat{q} \cdot \vec{x} - \omega t)}, \quad \text{and} \quad \vec{E}_2(\vec{x}, t) = \hat{e}_2 E_2 e^{i(q \hat{q} \cdot \vec{x} - \omega t)},$$

which also implies that

$$\vec{B}_l(\vec{x}, t) = \sqrt{\mu \varepsilon} \hat{q} \times \vec{E}_l(\vec{x}, t), \quad l = 1, 2.$$ 

These can be combined to write a most general homogeneous plane wave propagating along $\hat{q}$ direction as

$$\vec{E}(\vec{x}, t) = (\hat{e}_1 E_1 + \hat{e}_2 E_2) e^{i(q \hat{q} \cdot \vec{x} - \omega t)}.$$

where, in general the amplitudes $E_1$ and $E_2$ can be complex, which allows the possibility of a phase difference between waves of different linear polarizations.

- If $E_1$ and $E_2$ are of same phase, then above represents a linearly polarized plane wave with polarization vector $\hat{e}$ making an angle $\theta = \tan^{-1} (E_2/E_1)$ with $\hat{e}_1$, and magnitude $E = \sqrt{E_1^2 + E_2^2}$. 

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If $E_1$ and $E_2$ have different phases, then in general the wave is \textit{elliptically polarized}.

If $E_1$ and $E_2$ differ by $90^\circ$ in phase, then the wave is \textit{circularly polarized}, and described by

$$\mathbf{E}(\mathbf{x}, t) = (\hat{e}_1 \pm i \hat{e}_2) E_0 e^{i(q\mathbf{x} - \omega t)},$$

where $+$ sign corresponds to \textit{Left Circularly Polarized or LCP} and $-$ sign to \textit{Right Circularly Polarized or RCP} waves.