$$\vec{x} = \hat{n}\,x \simeq \hat{n}\,r$$

$$|x - x'| = \left(r^2 + x'^2 - 2r\,\hat{n}\cdot\vec{x}'\right)^{1/2}$$

$$\approx r - \hat{n}\cdot\vec{x}'$$

$$\frac{1}{|\vec{x}-\vec{x}'|} = \frac{1}{r} + \frac{\hat{n}\cdot\vec{x}'}{r^2} + \frac{1}{2}\frac{1}{r^3}\left(3(\hat{n}\cdot\vec{x}')^2 - x'^2\right) + \cdots$$

$$\frac{e^{|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} \cong \frac{e^{ikr}}{r}\left[1 + \left(\frac{1}{r} - ik\right)(\hat{n}\cdot\vec{x}') + \cdots\right]$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\,\frac{e^{ikr}}{r}\int \underline{\underline{\vec{J}(x')\,d^3x'}}$$

$$\begin{aligned}
\vec{\nabla}\cdot(x_i\vec{J}) &= (\nabla x_i)\cdot\vec{J} + x_i\,\vec{\nabla}\cdot\vec{J} \\
&= (\nabla x_i)\cdot\vec{J} - x_i\frac{\partial\rho}{\partial t} \\
&= (\nabla x_i)\cdot\vec{J} + i\omega x_i\rho. \\
&= J_i + i\omega x_i\rho.
\end{aligned}$$

$$\begin{aligned}
\int \vec{J}_i(x')\,d^3x_i &= \int \nabla\cdot(x_i'\vec{J})\,d^3x' \\
&\quad - i\omega\int \rho\, x_i'\,d^3x' \\
&= -i\omega\,\vec{p}_i
\end{aligned}$$

$$\vec{A}(\vec{x}) = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r} i\omega\vec{p}.$$

$$= -\frac{1}{4\pi\epsilon_0}\frac{ik}{c}\frac{e^{ikr}}{r}\vec{p}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = -\frac{1}{4\pi\epsilon_0}\frac{ik}{c}\nabla\left(\frac{e^{ikr}}{r}\right)\times\vec{p}.$$

$$= -\frac{1}{4\pi\epsilon_0}\frac{ik}{c}\left(ik-\frac{1}{r}\right)\frac{e^{ikr}}{r}\hat{r}\times\vec{p}$$

$$\vec{H} = \frac{ck^2}{4\pi}\frac{e^{ikr}}{r}\left(1+\frac{i}{kr}\right)\vec{r}\times\vec{p}$$

$$\vec{H} = \frac{ck^2}{4\pi}\frac{e^{ikr}}{r}\left(1+\frac{i}{kr}\right)\hat{r}\times\vec{p}$$

$$\vec{E} = \frac{e^{ikr}}{4\pi\epsilon_0 r}\left[k^2\hat{r}\times(\vec{p}\times\hat{r}) - \frac{ik}{r}\left(1+\frac{i}{kr}\right)\left(3(\hat{r}\cdot\vec{p})\hat{r}-\vec{p}\right)\right.$$

Radiation approximation

$$\vec{H} = \frac{ck^2}{4\pi}\frac{e^{ikr}}{r}\hat{r}\times\vec{p}$$

$$\vec{E} = c\mu\,\vec{H}\times\hat{r}$$

$$\vec{S} = \frac{1}{2} R\ell\,(\vec{E} \times \vec{H}^*)$$

$$= \frac{\omega k c \mu}{8\pi} R\ell\left[(\vec{H} \times \hat{r}) \times H^*\right]$$

$$= \hat{r}\,\frac{\omega k c \mu}{8\pi} |H|^2.$$

$$= \hat{r}\,\frac{(\omega k)^2 \cdot c\mu}{32\pi^2} \frac{1}{r^2} (\vec{r} \times \vec{p})^2$$

$$= \hat{r}\,\frac{(\omega k)^2 c\mu}{32\pi^2} \cdot \frac{1}{r^2}\, p^2 \cdot \sin^2\theta$$

$$\frac{d\vec{P}}{d\Omega} = r^2\, \hat{r} \cdot \langle \vec{S} \rangle$$

$$= \frac{\omega^4 \cdot \mu}{32\pi^2 c}\, p^2 \sin^2\theta$$

$$P = \frac{\omega^4 \mu}{12\pi c}\, p^2$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{J}(x')\overline{\left(\frac{1}{r}-ik\right)}(\hat{n}\cdot\vec{x}')\,d^3x'$$

$$= \left[\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\overset{*}{\left(\frac{1}{r}-ik\right)}\right]\underline{\int \vec{J}(x')(\hat{n}\cdot\vec{x}')\,d^3x'}$$

$$= \left[\qquad * \qquad\right]\frac{1}{r}\int \vec{J}(\vec{x}')(\vec{x}\cdot\vec{x}')\,d^3x'$$

$$\Downarrow$$

$$\sum_{j=1}^{3}\int \vec{J}(\vec{x}')\,x_j x'_j\,d^3x'$$

$$\sum_{j=1}^{3} \int \vec{J}(\vec{x}')\, x_j\, x_j'\, d^3x'$$

$$\int \vec{\nabla}\cdot\left(x_j x_j' \vec{J} \right) d^3x' = 0$$

$$\int \left[\nabla (x_j x_j') \cdot \vec{J} + x_j x_j' \vec{\nabla}\cdot\vec{J} \right] = 0$$

$$\nabla (x_j x_j') =$$

$$\sum_{j=1}^{3} \int \vec{J}(x') \, x_j \, x_j' \, d^3x'$$

$$\int \nabla \cdot (x_i' x_j' \vec{J}) \, d^3x' = 0$$

$$\int \left[\nabla (x_i' x_j') \cdot \vec{J} + x_i' x_j' \nabla \cdot \vec{J} \right] d^3x' = 0$$

$$\Downarrow$$

$$\nabla (x_i' x_j') \cdot \vec{J} = x_j' J_i + x_i' J_j$$

$$\int \left(x_j' J_i + x_i' J_j + i\omega \rho' x_i' x_j' \right) d^3x' = 0$$

$$\sum_{j=1}^{3} \int J_i \, x_j \, x_j' = \sum_{j=1}^{3} x_j \int J_i \, x_j' \, d^3x'$$

$$= -\sum_{j=1}^{3} x_j \int \left(J_j \, x_i' + i\omega\rho' x_i' x_j' \right) d^3x'$$

$$= \frac{1}{2} \sum_{j=1}^{3} x_j \int \left(\overbrace{J_i \, x_j' - J_j \, x_i'} - i\omega\rho' x_i' x_j' \right) d^3x'$$