Prof D.K Ghosh
Lec [illegible]

$$k_x = \frac{\pi}{a}$$

$$k_z = \frac{\pi}{d}$$

$$\langle W \rangle = \frac{\epsilon}{2} \cdot \frac{\omega^2 \mu^2 a^2}{\pi^2} |H_z^0|^2 \times b \times \int_0^a \frac{1 - \cos 2k_x x}{2} \, dx \times \int_0^d \frac{1 - \cos(2k_z z)}{2} \, dz$$

$$= \frac{\epsilon}{2} \frac{\omega^2 \mu^2}{\pi^2} |H_z^0|^2 \times b \times \frac{a^3}{2} \times \frac{d}{2}$$

$$= \frac{\epsilon}{8} \frac{\omega^2 \mu^2}{\pi^2} |H_z^0|^2 \cdot a^3 b d$$

$$|J_s|^2 = |H_z^0|^2 \sin^2 k_z z$$

$$\text{Loss} = 2 \times \frac{1}{2} R_s \int |J_s|^2 \, dy \, dz$$

$$= R_s |H_{z0}|^2 \int_0^b dy \int_0^d \sin^2 k_z z \, dz$$

$$= R_s |H_{z0}|^2 \cdot \frac{bd}{2}$$

Side Surfaces

$$|J_s|^2 = |H_z^0|^2 \left[\frac{a^2}{d^2} \sin^2(k_x x) \cos^2(k_z z) + \cos^2(k_x x) \sin^2(k_z z) \right]$$

$$\text{Loss} = |H_z^0|^2 \left[\frac{a^3 \cancel{d}}{4d} + \frac{ad}{4} \right]$$

Top & Bottom:

$$= R_s \frac{a^2}{d^2} |H_z^0|^2 \cdot \frac{ab}{2}$$

$$e^{-\gamma z}$$

$$\frac{\partial}{\partial z} \sim -\gamma$$

$$\vec{\nabla} \times \vec{H} = i\omega\epsilon \vec{E}$$

$$i\omega\epsilon E_\rho = (\vec{\nabla} \times \vec{H})_\rho$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\varphi} H_z - \frac{\partial}{\partial z} H_\varphi$$

$$i\omega\epsilon E_\varphi = \frac{\partial}{\partial z} H_\rho - \frac{\partial}{\partial\rho} H_z$$

$$= -\gamma H_\rho - \frac{\partial}{\partial\rho} H_z$$

$$i\omega\epsilon E_z = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho H_\varphi) - \frac{1}{\rho}\frac{\partial}{\partial\varphi} H_z$$

$$i\omega\epsilon E_\rho = \frac{1}{\rho}\frac{\partial}{\partial\phi}H_z + \gamma H_\phi$$

$$-i\omega\mu H_\phi = -\gamma E_\rho - \frac{\partial}{\partial\rho}E_z$$

Eliminate H_ϕ

$$(\underbrace{\gamma^2 + \epsilon\mu\omega^2}) E_\rho = -\frac{i\mu\omega}{\rho}\frac{\partial}{\partial\phi}H_z - \gamma\frac{\partial}{\partial\rho}E_z$$

$$\gamma^2 + \epsilon\mu\omega^2 = k^2$$

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z$$

$$\cancel{H_z(x,y,z) = X(x)\,Y(y)\,Z(z)}$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}\right) H_z + \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2} H_z + \frac{\partial^2}{\partial z^2} H_z$$

$$= -\omega^2 \mu \epsilon H_z$$

$$H_z(\rho, \phi, z) = R(\rho)\,F(\phi)\,Z(z)$$

$$\frac{1}{R}\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}\right) R + \frac{1}{F}\frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2} F$$

$$+ \frac{1}{Z}\frac{\partial^2}{\partial z^2} Z = -\omega^2 \mu \epsilon$$

$$\frac{1}{R}\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)R+\frac{1}{F}\cdot\frac{1}{\rho^2}\cdot\frac{\partial^2}{\partial\phi^2}F+\omega^2\mu\epsilon$$

$$=-\frac{1}{Z}\frac{\partial^2}{\partial z^2}Z(z)$$

$$=-\gamma^2$$

$$Z(z)\sim e^{-\gamma z}$$

$$\frac{\frac{\rho}{R}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial R}{\partial\rho}\right) + (\omega^2\mu\epsilon + \gamma^2)\rho^2}{} = -\frac{1}{F}\frac{\partial^2}{\partial\phi^2}F$$

$$= n^2.$$

$$\frac{\partial^2 F}{\partial\phi^2} + n^2 F = 0$$

$$F \sim A\,\mathrm{Cos}(n\phi) + B\,\mathrm{Sin}(n\phi)$$

$$\phi \rightarrow \phi + 2m\pi$$

n integer.

$$\frac{\rho}{R}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}R\right)+(\underbrace{\omega^2\mu\epsilon+\gamma^2}_{k_\rho^2})\rho^2-n^2=0$$

$k_\rho \cdot \rho$

$$\frac{\partial^2}{\partial(k_\rho\rho)^2}\cdot R+\frac{1}{(k_\rho\rho)}\frac{\partial}{\partial(k_\rho\rho)}R$$

$$+\left[1-\frac{n^2}{(k_\rho\rho)^2}\right]=0$$

$$\frac{d^2}{dx^2}y+\frac{1}{x}\frac{\partial}{\partial x}y+\left(1-\frac{n^2}{x^2}\right)=0$$

Bessel Equation.

J_n, N_n — Neumann Fn