Prof. D. Ghosh
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Lec. 36

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\sim e^{i\omega t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \nabla(\vec{\nabla}.\vec{E}) - \nabla^2 \vec{E}$$

$$= -\mu(i\omega)(\nabla \times \vec{H})$$

$$= -\mu(i\omega)[\sigma \vec{E} + \epsilon i\omega \vec{E}]$$

$$\nabla^2 \vec{E} = i\mu\omega(\sigma + i\epsilon\omega)\vec{E}$$

$$\nabla^2 \vec{H} = i\omega\mu(\sigma + i\epsilon\omega)\vec{H}$$

$$\sigma = 0$$

$$\vec{\nabla} \times \vec{H} = i\omega\epsilon\vec{E}$$

$$\frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y = i\omega\epsilon E_x$$

$$\frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z = i\omega\epsilon E_y$$

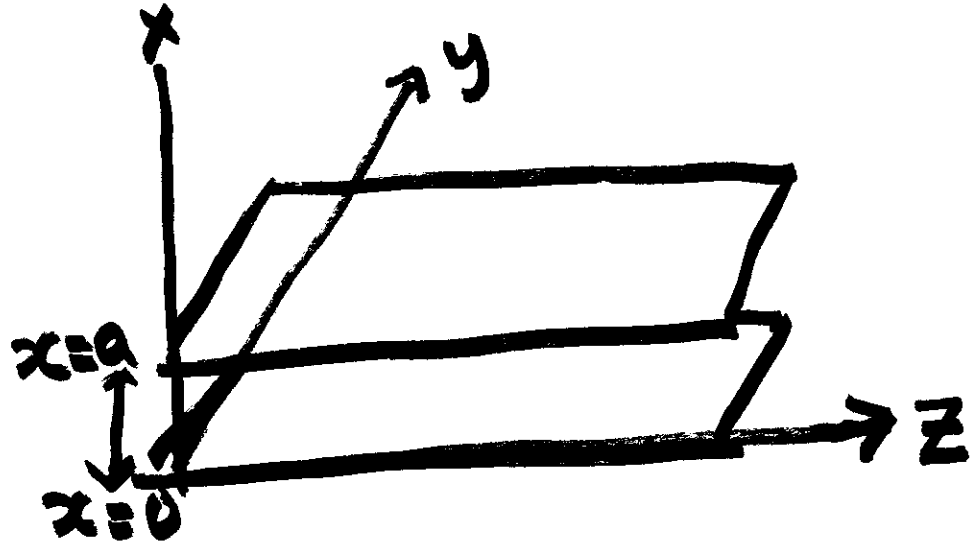
$$\frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x = i\omega\epsilon E_z$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y = -i\mu\omega H_x$$

$$\frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_z = -i\mu\omega H_y$$

$$\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x = -i\mu\omega H_z$$



$\gamma = \alpha + i\beta$

Propagati

$$\vec{E}(x,y,z) = E(x,y)\, e^{i\omega t - \gamma z}$$

$$\frac{\partial}{\partial z} \rightarrow -\gamma.$$

$$\frac{\partial}{\partial y} \rightarrow 0$$

$$\left.\begin{aligned}
\gamma H_y &= i\omega\epsilon E_x \\
-\gamma H_x - \frac{\partial}{\partial x} H_z &= i\omega\epsilon E_y \\
\frac{\partial}{\partial x} H_y &= i\omega\epsilon E_z .
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\gamma E_y &= -i\omega\mu H_x \\
-\gamma E_x - \frac{\partial}{\partial x} E_z &= -i\omega\mu H_y \\
\frac{\partial}{\partial x} E_y &= -i\omega\mu H_z
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\nabla^2 \vec{E} &= -\omega^2\mu\epsilon \vec{E} \\
\nabla^2 \vec{H} &= -\omega^2\mu\epsilon \vec{H}
\end{aligned}\right\} \left(\frac{\partial^2}{\partial x^2} + \gamma^2\right)\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = -\omega^2\mu\epsilon \begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix}$$

TE - Mode $E_z = 0$;

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2 E_y = -\omega^2 \mu \epsilon E_y .$$

$$k^2 = \gamma^2 + \omega^2 \mu \epsilon$$

$$\frac{\partial^2 E_y}{\partial x^2} + k^2 E_y = 0$$

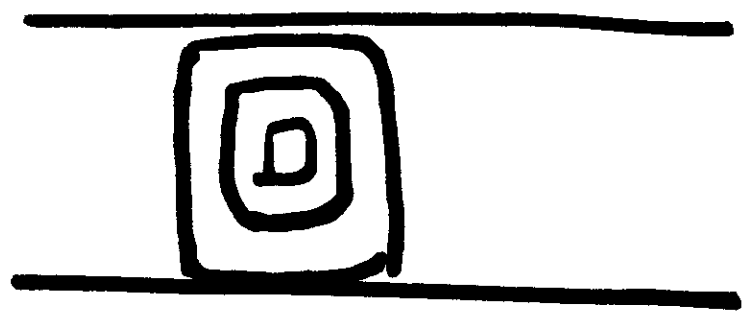
$$E_y = A \sin kx + B \cos kx .$$

At $x = 0$ $E_y = 0$

$$E_y = E_y^0 \sin\left(\frac{n\pi}{d} x\right) e^{-\gamma z} \qquad n = 1, 2, 3 ..$$

$$H_x = \frac{1}{i\omega\mu}\frac{\partial E_y}{\partial z} = -E_y^0 \frac{\gamma}{i\omega\mu} \sin\left(\frac{n\pi}{d}x\right) e^{-\gamma z}$$

$$H_z = -E_y^0 \cdot \frac{n\pi}{i\omega\mu d} \cdot \cos\left(\frac{n\pi x}{d}\right) e^{-\gamma z}$$



$$\gamma = \sqrt{\left(\frac{n\pi}{d}\right)^2 - \omega^2\mu\epsilon} = i\beta$$

If $\omega > \omega_c = \frac{1}{\sqrt{\mu\epsilon}}\frac{n\pi}{d}$ Propagating Soln.

$\omega < \omega_c \rightarrow$ Attenuating

$$v_\phi = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2\mu\epsilon - \left(\frac{n\pi}{d}\right)^2}} \rightarrow \infty$$

as $\omega \rightarrow \omega_c$.

$\omega \gg$ $\qquad v_\phi = \frac{1}{\sqrt{\mu\epsilon}} \rightarrow c$ for vacuum

$$\frac{\partial}{\partial y} H_z + \gamma H_y = i\omega\epsilon E_x$$

$$-\gamma E_x - \frac{\partial}{\partial x} E_z = -i\omega\mu H_y$$

$$i\omega\epsilon E_x = \gamma H_y + \frac{\partial}{\partial y} H_z.$$

$$= \frac{\gamma}{i\omega\mu}\left[E_x + \frac{\partial}{\partial x} E_z\right] + \frac{\partial}{\partial y} H_z$$

$$\left(i\omega\epsilon - \frac{\gamma}{i\omega\mu}\right) E_x = \frac{\gamma}{i\omega\mu}\frac{\partial}{\partial x} E_z + \frac{\partial}{\partial y} H_z$$

$$E_x = \frac{-\gamma}{k^2}\frac{\partial}{\partial x} E_z - \frac{i\omega\mu}{k^2}\frac{\partial}{\partial y} H_z$$

$$k^2 = \gamma^2 + \omega^2\mu\epsilon$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right) \tilde{H}_z(x,y) = 0$$

$$H_z(x,y,z) = \tilde{H}_z(x,y)\, e^{-\gamma z} .$$

$$\tilde{H}_z(x,y) = X(x)\, Y(y)$$

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + k^2 XY = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + k^2 = -\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = k_y^2$$