1.

25/10/11
Prof. D. [illegible]
Lec. 33

$$\vec{l} = \vec{r} \times \vec{p}$$

$$\frac{\partial}{\partial t}\vec{l} = \vec{r} \times \left(\rho \vec{E} + \vec{J} \times \vec{B}\right)$$

$$\frac{\partial}{\partial t}\vec{l} + \frac{1}{c^2}\vec{r} \times \vec{S} = \epsilon_0 \vec{r} \times \left[\vec{E}(\vec{\nabla}\cdot\vec{E}) - E \times (\nabla \times E)\right] + \frac{1}{\mu_0}\vec{r} \times \left[\vec{B}(\vec{\nabla}\cdot\vec{B}) - B \times (\nabla \times E)\right]$$

$$\frac{\partial}{\partial t}\left(\vec{l}_{mech} + \vec{l}_{em}\right) = \vec{r} \times \left(\vec{\nabla}\cdot\overleftrightarrow{T}\right)$$

$$\frac{d}{dt}\left(\vec{L}_{mech} + \int_{vol} \vec{\ell}_{em}\, d^3x\right) = \int_{surface} \left(\vec{r} \times \overleftrightarrow{T}\right) \cdot d\vec{S}$$

$$\vec{L} = \int \left(\vec{r} \times \frac{\vec{S}}{c^2}\right) 2\pi r\, dr$$

$$\vec{E} = -\frac{\lambda}{2\pi\epsilon_0 r}\hat{r} \quad (r<a)$$

$$\vec{p} = \frac{1}{\mu_0 c^2}\vec{E}\times\vec{B} = \frac{1}{\mu_0 c^2}\left(-\frac{\lambda}{2\pi\epsilon_0 r}\right)\hat{r}\times B_0\hat{z}$$

$$= \frac{B_0\lambda}{2\pi r}\hat{\phi}$$

$$\vec{L} = \frac{B_0\lambda}{2\pi}\int_0^a \frac{\vec{r}\times\hat{\phi}}{r} 2\pi r\, dr = \frac{B_0\lambda}{\cancel{2\pi}}\frac{a^2}{2}\hat{z}$$

$$\vec{L} = \frac{B_0\lambda a^2}{2}\hat{z}$$

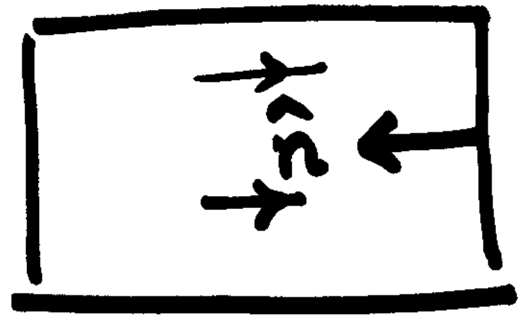
$$J_\phi = \frac{Q}{T} = 2\pi a \sigma \times \frac{\omega}{2\pi} = \frac{\lambda \omega}{2\pi}$$

$$\vec{B}_f = \mu_0 J \hat{z} = \mu_0 \frac{\lambda\omega}{2\pi} \hat{z}.$$

$$\vec{L}_f = \frac{B_f \lambda a^2}{2} \hat{z}$$

$$= \frac{\mu_0}{4\pi} \lambda^2 a^2 \omega.$$

$$I\omega + \frac{\mu_0}{4\pi}\lambda^2 a^2 \omega = \frac{B_0 \lambda a^2}{2}$$



$$d\vec{F} = \overleftrightarrow{T} \cdot d\vec{S}$$

$$T_{xx} = \epsilon_0 \left(E_x^2 - \frac{1}{2} E^2 \right) + \frac{1}{\mu_0} \left(B_x^2 - \frac{1}{2} B^2 \right)$$

$$E_x^2 = \frac{1}{3} E^2 \qquad B_x^2 = \frac{1}{3} B^2$$

$$dF_x = \frac{1}{6} \epsilon_0 E^2 + \frac{1}{6\mu_0} B^2$$

$$= \frac{1}{3} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0} \right) = \frac{u}{3}$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$\nabla(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\mu \epsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2}.$$

$$\vec{E} = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} \mp \omega t)}$$

$$\vec{B} = \vec{B}_0\, e^{i(\vec{k}\cdot\vec{r} \mp \omega t)}$$

$$\vec{k}\cdot\vec{r} - \omega t = \text{Constant} \Rightarrow \xi = \vec{k}\cdot\vec{r} = \text{Cons}$$

$$-\omega t + |k|\,\xi = \cancel{\text{Const}}\ \text{Const}$$

$$|k|\frac{d\xi}{dt} = \omega; \quad \frac{d\xi}{dt} = \frac{\omega}{k} = v.$$

$$\vec{\nabla}\cdot\vec{E} = 0 \Rightarrow \vec{k}\cdot\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \vec{k}\cdot\vec{B} = 0$$

$$\nabla \times \vec{B} = \mu\epsilon \frac{\partial \vec{E}}{\partial t} \qquad \nabla \rightarrow ik \qquad \frac{\partial}{\partial t} \mapsto -i\omega$$

$$i\vec{k}\times\vec{B} = -i\omega\mu\epsilon\vec{E}$$

$$\vec{k}\times(\vec{k}\times\vec{B}) = -\omega\mu\epsilon\,\vec{k}\times\vec{E}$$

$$\vec{k}\,\cancel{(\vec{k}\cdot\vec{B})} - \vec{B}k^2 = -\omega\mu\epsilon\,\vec{k}\times\vec{E}$$

$$\vec{B} = \frac{\omega\mu\epsilon}{k^2}\vec{k}\times E$$

$$= \frac{1}{k}(\omega\mu\epsilon)\,\hat{k}\times\vec{E}$$

$$= \frac{\omega}{k}\cdot\frac{1}{v^2}\,\vec{k}\times\vec{E} = \frac{1}{v}\,\vec{k}\times\vec{E}$$

$$\boxed{\mu\epsilon = \frac{1}{v^2}}$$

$$\vec{k}\cdot\vec{E} = 0$$

$$\vec{k}\cdot\vec{B} = 0$$

$$\vec{B} = \frac{1}{v}\,\vec{k}\times\vec{E}$$

$\{\vec{E}, \vec{B}, \hat{k}\}$ Right Handed Triad

$$u = \frac{1}{2}\epsilon E^2 + \frac{B^2}{2\mu}$$

$$= \frac{1}{2}\epsilon\left[E^2 + \frac{B^2}{\mu\epsilon}\right]$$

$$= \frac{1}{2}\epsilon\left[E^2 + \underline{v^2 B^2}\right]$$

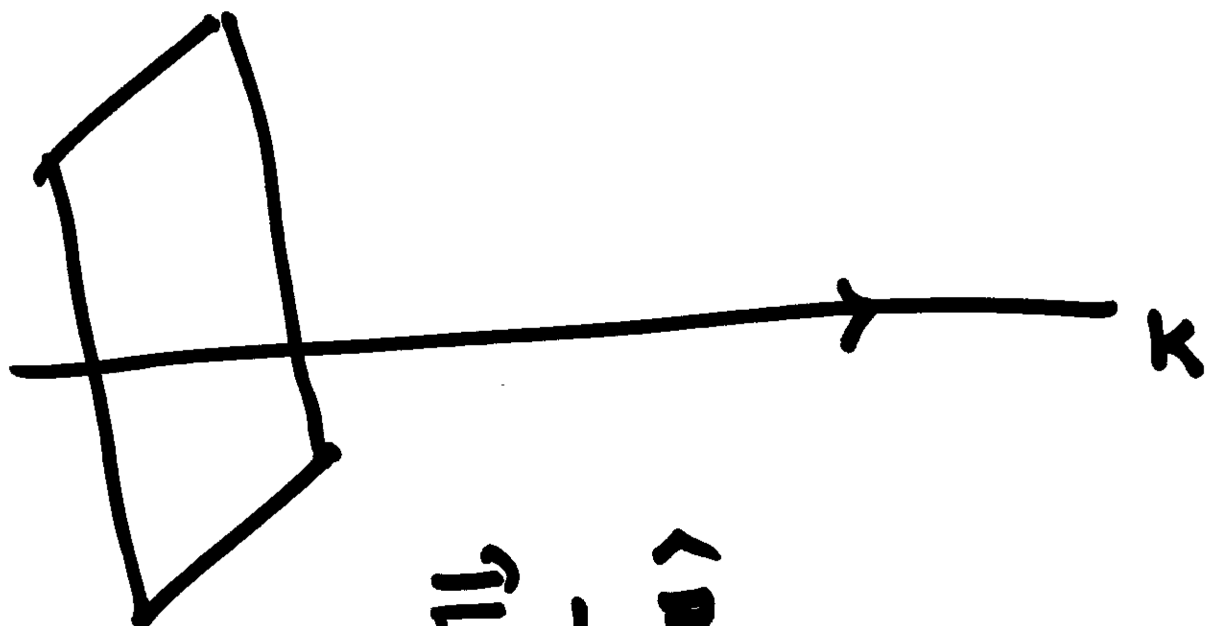
$$= \epsilon E^2$$

$$|\vec{S}| = |\vec{E}\times\vec{H}| = \frac{EB}{\mu_0} = c\epsilon_0 E^2$$

$$\vec{S} = c\epsilon_0 E_0^2 \underline{\cos^2(kz-\omega t)}\,\hat{k}$$

$$I = \langle\vec{S}\rangle = \frac{1}{2}c\epsilon_0 E_0^2\,\hat{k}$$

Polarization



$$\vec{E} \perp \hat{z}$$

$$\vec{E}(z,t) = \mathrm{Re}\left[\left(\underline{E_{ox}}\,\hat{i} + \underline{\underline{E_{oy}}}\,\hat{j}\right) e^{-i\omega t + ikz}\right]$$

$$E_{0x} = |E_{0x}| e^{i\varphi}$$

$$E_{0y} = |E_{0x}| e^{i\varphi}$$

$$\vec{E} = \left(|E_{0x}| \hat{i} + |E_{0y}| \hat{j} \right) \cos(kz - \omega t + \varphi)$$

Linearly polarized

$$E_{0x} = |E_{0x}|$$

$$E_{0y} = |E_{0y}| e^{i\varphi}$$

Elliptically polarized

$\varphi = \frac{\pi}{2}$ $\quad |E_{0x}| = |E_{0y}|$ $\quad \odot$arly