

$$\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I.$$

$$\oint \mathbf{B} \cdot d\mathbf{L} = 0$$

$$\Phi_E = \int \vec{D} \cdot d\vec{S}$$

$$\frac{d\Phi_E}{dt} = \frac{d}{dt} \int \vec{D} \cdot d\vec{S}$$

$$= \frac{d}{dt} \int \nabla \cdot \vec{D} d^3r$$

$$= \frac{d}{dt} \int \rho d^3r$$

$$= \frac{dQ}{dt}$$

$$\boxed{i_a = \frac{d\Phi_E}{dt}}$$

$$\begin{aligned}\Phi_E &= \int \vec{E} \cdot d\vec{S} = E \cdot A \\ &= \frac{Q}{\epsilon_0 A} \cdot A = \frac{Q}{\epsilon_0}.\end{aligned}$$

~~$$I_d = \frac{1}{\epsilon_0} \frac{dQ}{dt}.$$~~

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt}.$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left( I + \epsilon_0 \frac{\partial \Phi}{\partial t} \right)$$

$$\int (\nabla \times \vec{B}) \cdot d\vec{S} = \mu_0 \left( \int \vec{J} \cdot d\vec{S} + \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{S} \right)$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \underline{\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$$\begin{aligned} \mu_0 \vec{J} &= \nabla \times \vec{M} \\ \mu_0 \vec{J} &= \frac{\partial \vec{P}}{\partial t} \end{aligned}$$

$$\vec{J} = \mu_0 \left( \underbrace{\vec{J}_{free} + \nabla \times \vec{M} + \frac{\partial \vec{P}}{\partial t}}_{+ \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}} \right)$$

$$\nabla \times \vec{B} = \mu_0 \left( \vec{J}_f + \nabla \times \vec{M} + \frac{\partial \vec{P}}{\partial t} \right) + \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\begin{aligned} \nabla \times \left( \frac{\vec{B}}{\mu_0} - \vec{M} \right) &= \vec{J}_f + \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t} \\ &= \vec{J}_f + \frac{\partial}{\partial t} \left( \epsilon_0 \vec{E} + \vec{P} \right) \end{aligned}$$

$$\begin{aligned} \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ &= -\mu_0 \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \end{aligned}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_{f+e}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

4 Eqs. in  
6 quantities

$E_x, E_y, E_z$

$B_x, B_y, B_z$ .

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

Linear Electric/Magnetic material <sup>7</sup>

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{E} = -\nabla V \quad \text{— electrostatics.}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial (\nabla \times \vec{A})}{\partial t}$$

$$\nabla \times [\nabla V + \nabla \times \vec{A}] =$$

$$\nabla \times \left[ -\nabla V + \frac{\partial \vec{A}}{\partial t} \right] = 0 \quad \text{— e.s.}$$

$$\nabla \times \left[ \vec{E} + \frac{\partial \vec{A}}{\partial t} \right] = 0$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$



$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla^2 V + \frac{\partial}{\partial t} (\nabla \cdot \vec{A}) = -\frac{\rho}{\epsilon_0}$$

Gauss' Law  
Electrostat.

Faraday's  
Law.

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$= \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left( -\nabla V - \frac{\partial \vec{A}}{\partial t} \right)$$

$$\left. \mu_0 \epsilon_0 = \frac{1}{c^2} \right\} \left( \nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} \right) - \nabla \left( \nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} \right)$$

$$= 0$$

$$\nabla^2 V + \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{A}) = -\frac{\rho}{\epsilon_0}$$

$$\left( \nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} \right) - \nabla \left( \underbrace{\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \frac{\partial V}{\partial t}}_{=0} \right) = \cancel{-\mu_0 \vec{J}}$$

$$\vec{A} \rightarrow A' = A + \nabla \psi$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

$$V \rightarrow V' = V - \frac{\partial \psi}{\partial t}$$

$$\nabla^2 V - \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} = -\rho / \epsilon_0 \quad \parallel$$

$$\nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} \quad \parallel$$

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} = 0 \quad \text{Lorentz Gauge.}$$

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} = f(\vec{r}, t) \neq 0$$

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \nabla \psi$$

$$V \rightarrow V' = V - \frac{\partial \psi}{\partial t}$$

$$\begin{aligned}
 W_{el} &= \frac{1}{2} \int d^3x \rho(x) \varphi(x) \\
 &= \frac{1}{2} \int d^3x \varphi(x) (\vec{\nabla} \cdot \vec{D}) \\
 &= -\frac{1}{2} \int d^3x \vec{D} \cdot (\nabla \varphi) \\
 &= \frac{1}{2} \int (\vec{D} \cdot \vec{E}) d^3x
 \end{aligned}$$

$$\nabla \cdot (\varphi \vec{D})$$

$$\cancel{u_E} \quad u_E = \frac{\vec{D} \cdot \vec{E}}{2}$$

$$\begin{aligned}
 W_{\text{mag}} &= \frac{1}{2} \int \vec{A} \cdot \vec{J} \, d^3x \\
 &= \frac{1}{2} \int \vec{A} \cdot (\nabla \times \vec{H}) \, d^3x \\
 &= \frac{1}{2} \int \vec{H} \cdot (\nabla \times \vec{A}) \, d^3x \\
 &= \frac{1}{2} \int \vec{B} \cdot \vec{H} \, d^3x
 \end{aligned}$$

$$u_{\text{mag}} = \frac{\vec{B} \cdot \vec{H}}{2} = \frac{|\vec{B}|^2}{2\mu}$$

$$u_{\text{elec}} = \frac{\vec{E} \cdot \vec{D}}{2} = \frac{\epsilon}{2} |\vec{E}|^2$$

$$u = \frac{\epsilon}{2} |\vec{E}|^2 + \frac{1}{2\mu} |\vec{B}|^2$$

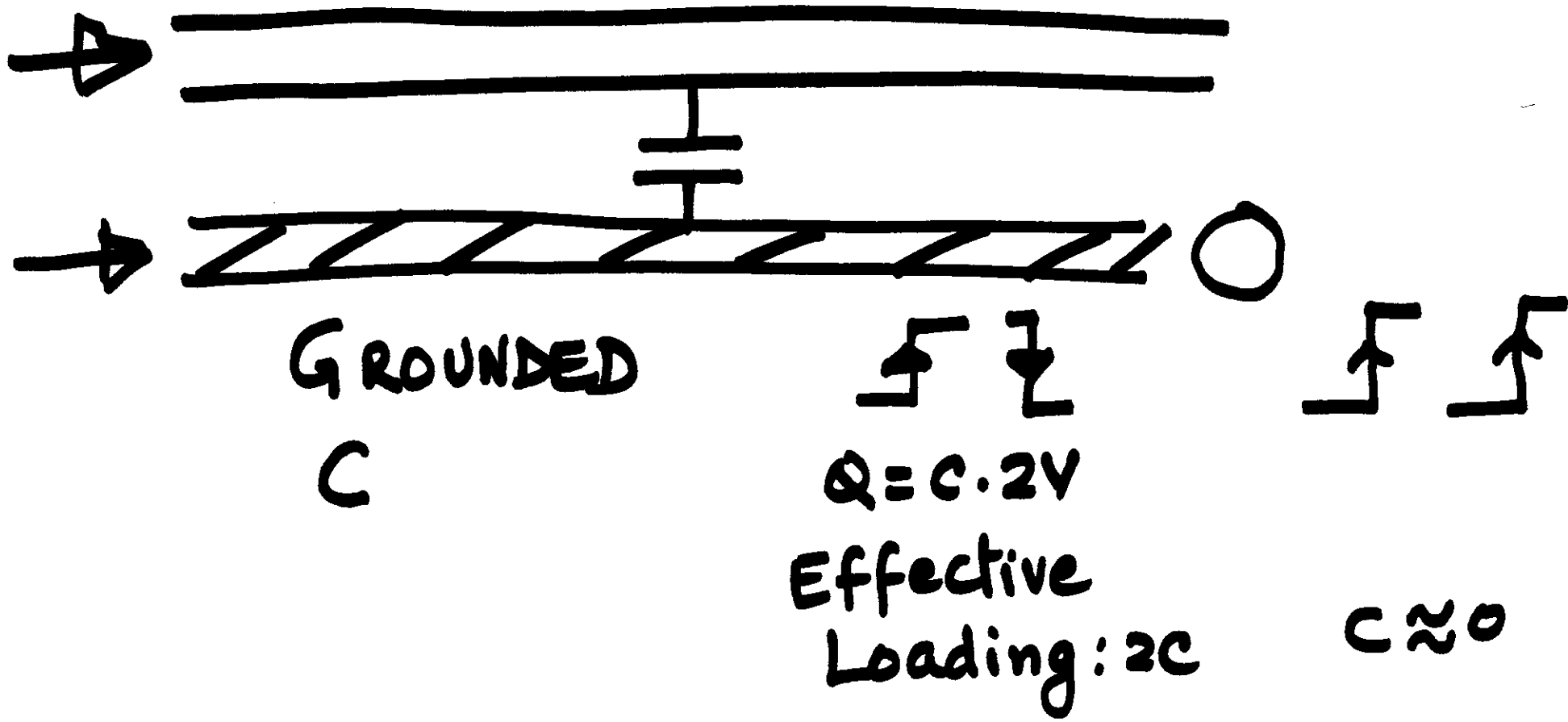
$$\begin{aligned}
 P_{\text{mech}} &= \int \vec{E} \cdot \vec{v} \, d^3x \\
 &= \int \rho (\vec{E} * \vec{v} \times \vec{B}) \cdot \vec{v} \, d^3x \\
 &= \int \vec{E} \cdot \vec{J} \, d^3x
 \end{aligned}$$

$$u = \frac{\epsilon}{2} |\vec{E}|^2 + \frac{1}{2\mu} |\vec{B}|^2$$

$$\begin{aligned}
 \frac{dW}{dt} &= \frac{1}{2} \int \left( 2\epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} + \frac{2}{\mu} \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} \right) d^3x \\
 &= \int \epsilon \vec{E} \cdot \left( \frac{1}{\epsilon} \nabla \times \vec{H} - \frac{1}{\epsilon} \vec{J} \right) + \frac{2}{\mu} \vec{H} \cdot (-\nabla \times \vec{E}) \\
 &= \int \left[ \vec{E} \cdot (\nabla \times \vec{H}) - \vec{H} \cdot (\nabla \times \vec{E}) \right] d^3x - \int \vec{E} \cdot \vec{J} \, d^3x \\
 &= -\int \nabla \cdot (\vec{E} \times \vec{H}) \, d^3x - \int \vec{E} \cdot \vec{J} \, d^3x
 \end{aligned}$$

$$\frac{\partial u}{\partial t} + \nabla \cdot \vec{S} = -\vec{E} \cdot \vec{J}.$$

$$\vec{S} = \vec{E} \times \vec{H} \quad \text{Poynting Vector.}$$



Data dependent