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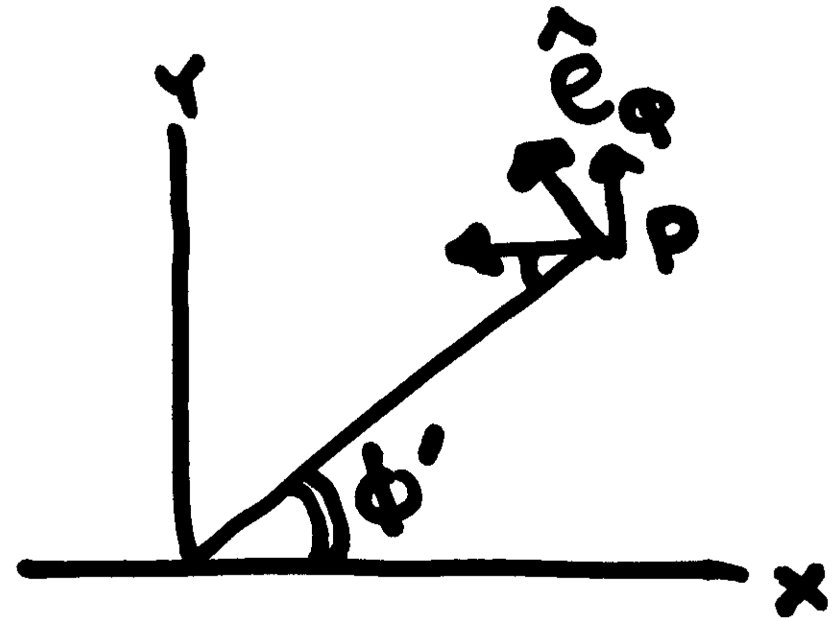
28 (13/10/11)

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

$$\vec{A}(\vec{r}) = -\frac{\mu_0}{4\pi}\int_{\text{Surface}}\frac{\hat{n}'\times\vec{M}(r')}{|\vec{r}-\vec{r}'|}\,ds$$

$$J_s(\vec{r}') = -\hat{n}'\times\vec{M}(r')$$
$$= M\sin\theta\,\hat{e}_\phi$$



$$\hat{e}_\varphi = \left[-\sin\varphi' \,\hat{i} + \cos\varphi' \,\hat{j} \right]$$

$$\vec{J}_S(\vec{r}') = M \sin\theta' \left[-\sin\varphi' \,\hat{i} + \cos\varphi' \,\hat{j} \right]$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{M \sin\theta' (-\sin\varphi' \, \hat{i} + \cos\varphi' \, \hat{j})}{|\vec{r} - \vec{r}\,'|} \, dS$$

$$\sin\theta' \cos\varphi'$$

$$Y_{1,1}(\theta', \varphi') = -\sqrt{\frac{3}{8\pi}} \sin\theta' e^{i\varphi'}$$

$$Y_{1,-1}(\theta', \varphi') = +\sqrt{\frac{3}{8\pi}} \sin\theta' e^{-i\varphi'}$$

$$\sin\theta' \cos\varphi' = -\sqrt{\frac{2\pi}{3}} \left[Y_{11}(\theta', \varphi') - Y_{1,-1}(\theta', \varphi') \right]$$

$$\sin\theta' \sin\varphi' = \sqrt{\frac{2\pi}{5}} \, i \left[Y_{11}(\theta', \varphi') + Y_{1,-1}(\theta', \varphi') \right]$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = 4\pi \sum_{\ell} \sum_{m} \frac{1}{(2l+1)} \frac{r_<^{\ell}}{r_>^{\ell+1}} Y^*_{\ell m}(\theta',\varphi') Y_{\ell m}(\theta,\varphi)$$

Choose $\varphi = 0$

$$A_y(\vec{r}) = \frac{\mu_0}{4\pi} \sqrt{\frac{2\pi}{5}} MR^2 \cdot 4\pi \int \sum_{\ell,m} \frac{1}{(2l+1)} \frac{r_<^{\ell}}{r_>^{l+1}}$$

$$\times \left(Y_{1,-1} - Y_{1,1}\right) Y^*_{lm}(\theta',\varphi') Y_{lm}(\theta,0) \, d\Omega'$$

$$= MR^2 \sqrt{\frac{2\pi}{3}} \frac{\mu_0}{3} \frac{r_<}{r_>^2} \left(Y_{1,-1}(\theta,0) - Y_{1,1}(\theta,0)\right)$$

$2\sin\theta$.

$$A_y = MR^2 \frac{2\mu_0}{3} \sin\theta \cdot \frac{r_<}{r_>^2}$$

Inside $r_> = R$

$$A_y = \cancel{MR^2}\, M \frac{2\mu_0}{3} \underline{\underline{r \sin\theta}} = M \frac{2\mu_0}{3} x$$

Outside

$$A_y = MR^3 \cdot \frac{2\mu_0}{3} \cdot \frac{\sin\theta}{r^2}.$$

$\vec{B}$ inside

$$B_z = \frac{\partial A_y}{\partial x} = M.\frac{2\mu_0}{3}$$

$$\vec{B} = \frac{2}{3}\mu_0\vec{M} + \vec{B}_0$$

$$\vec{H} = \frac{1}{\mu_0}\left[B_0 + \frac{2}{3}\mu_0\vec{M}\right] - \vec{M}$$

$$= \frac{1}{3}\vec{M}$$

$\vec{B} = \mu\vec{H}$ Paramagnetic

$$\frac{2}{3}\mu_0\vec{M} + \vec{B}_0 = \mu\left(\frac{B_0}{\mu_0} - \frac{1}{3}\vec{M}\right)$$

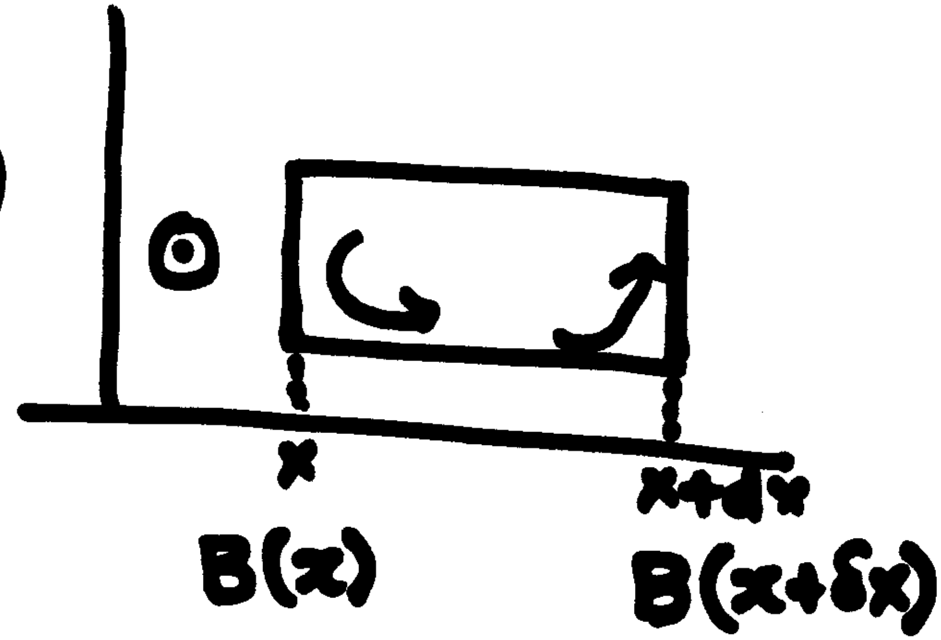
$$\vec{M} = \frac{3}{\mu_0}\left(\frac{\mu-\mu_0}{\mu+2\mu_0}\right)\vec{B}_0$$

$$\vec{B} = \frac{3\mu}{\mu+2\mu_0}\vec{B}_0$$

$$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot \vec{dl}$$

$$= \int (\vec{v} \times \vec{B}(x)) \cdot (-\hat{j}\, \delta y)$$

$$+ \int (\vec{v} \times \vec{B}(x + \delta x)) \cdot (+\hat{j}\, \delta y)$$



$$\vec{B}(x + \delta x) = B(x) + \frac{\partial B}{\partial x} \delta x$$

$$\mathcal{E} = -\int \left(\vec{v} \times \frac{\partial \vec{B}}{\partial x} \right) \delta x \, \delta y$$

$$= -\int \frac{\partial B}{\partial t} \, ds = -\frac{d}{dt} \phi_B$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int \vec{B} \cdot \vec{ds} .$$

$$\int (\vec{\nabla} \times \vec{E}) \cdot \vec{ds} = -\frac{d}{dt}\int \vec{B} \cdot \vec{ds}$$

$$= -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

$$\left.\begin{aligned} \nabla \cdot \vec{E} &= \rho/\epsilon_0 \\ \nabla \cdot \vec{B} &= 0 \end{aligned}\right\}$$