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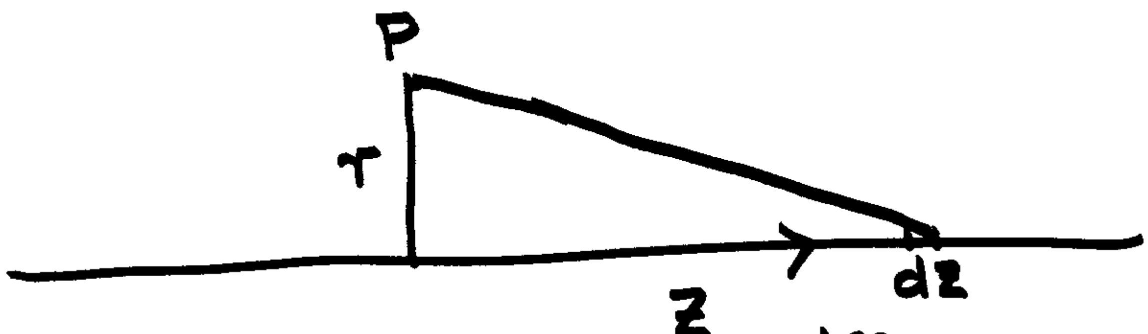
$$\vec{\nabla} \times \vec{A} = \vec{B}$$

$$\vec{\nabla} \cdot \vec{A} = 0 \quad \text{Coulomb Gauge.}$$



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} I \hat{k} \int_{-\infty}^{+\infty} \frac{dz}{\sqrt{r^2 + z^2}}.$$

$$\int \frac{dz}{\sqrt{z^2+r^2}} \qquad \begin{aligned} z &= r\tan\theta \\ dz &= r\sec^2\theta \cdot d\theta \end{aligned}$$

$$\int \frac{r\sec^2\theta \, d\theta}{r\sec\theta} = \int \sec\theta \, d\theta$$

$$= \ln\left|\tan\theta + \sec\theta\right|$$

$$\vec{A} = \frac{\mu_0 I \hat{k}}{4\pi} \ln\left(z + \sqrt{r^2+z^2}\right)\Big|_{-\infty}^{+\infty} \rightarrow \infty$$

$$\vec{B}(r) = \frac{\mu_0 I}{2\pi r}\hat{\phi} = (\vec{\nabla} \times \vec{A})_\phi$$

$$\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} = \frac{\mu_0 I}{2\pi r}.$$

$$-\frac{\partial A_z}{\partial r} = \frac{\mu_0 I}{2\pi r} \quad \|$$

$$\vec{A}\,(\vec{r}) = \hat{k}\,\frac{\mu_0 I}{2\pi}\ln r + \nabla\Psi$$

$$\oint \vec{A}\cdot d\vec{l} = \int (\vec{\nabla}\times\vec{A})\cdot ds$$

$$= \int \vec{B}\cdot d\vec{s} = \phi_B$$

Inside Solenoid

$$\vec{B} = \mu_0 n I \hat{z}$$

$$\phi_B = \pi r^2 B = \pi r^2 \mu_0 n I.$$

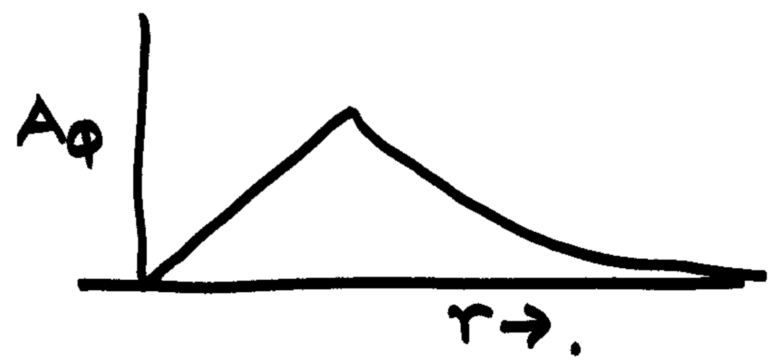
$$|A| 2\pi r = A_\phi . 2\pi r = \pi r^2 \mu_0 n I$$

$$A_\phi = \left(\frac{\mu_0 n I}{2}\right) r. \quad r \le R$$

Outside Solenoid $(r > R)$

$$\oint A \cdot dl = 2\pi r A_\phi$$
$$= \pi R^2 \times \mu_0 n I.$$

$$A_\phi = \frac{\mu_0 n I}{2r} R^2.$$



$$\vec{\nabla} \cdot \vec{A} = 0$$

$$\vec{A} = \frac{\vec{B} \times \vec{r}}{2}$$

$$\nabla \times (\vec{U} \times \vec{V}) = \vec{U}(\vec{\nabla} \cdot \vec{V}) - \vec{V}(\vec{\nabla} \cdot U) + (\vec{V} \cdot \vec{\nabla})\vec{U} - (\vec{U} \cdot \vec{\nabla})\vec{V}$$

$$\nabla \times \left(\frac{\vec{B} \times \vec{r}}{2} \right) = \frac{1}{2} \left[\vec{B}(\vec{\nabla} \cdot \vec{r}) - \cancel{\vec{r}(\vec{\nabla} \cdot \vec{B})} + \cancel{(\vec{r} \cdot \vec{\nabla})\vec{B}} - (\vec{B} \cdot \vec{\nabla})\vec{r} \right]$$

$$(\vec{B} \cdot \vec{\nabla})\vec{r} = \left(B_x \frac{\partial}{\partial x} + B_y \frac{\partial}{\partial y} + B_z \frac{\partial}{\partial z} \right)(\hat{i} x + \hat{j} y + \hat{k} z)$$

$$= \hat{i} B_x + \hat{j} B_y + \hat{k} B_z = \vec{B}.$$

$$\vec{A} = \frac{\vec{B} \times \vec{r}}{2}$$

$\vec{\nabla} \cdot \vec{A} = 0$ $\quad$ $\vec{B}$ in z-direction

$$\vec{A} = \left(-\frac{By}{2}, \frac{Bx}{2}, 0 \right)$$

$$= \left(-By, 0, 0 \right)$$

$$(\nabla \times \vec{A})_z = \left(\frac{\partial}{\partial x} By - \frac{\partial}{\partial y} Bx \right)$$

$$\vec{B} = \begin{cases} -\frac{\mu_0}{2} K \hat{j} & z > 0 \leftarrow \\ +\frac{\mu_0}{2} K \hat{j} & z < 0 \end{cases}$$

For $z > 0$

$$\vec{A}(x, y, z) = \frac{1}{2} \vec{B} \times \vec{r}$$

$$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -\frac{\mu_0 K}{2} & 0 \\ x & y & z \end{vmatrix}$$

$$= \frac{-1}{4} \mu_0 K z \, \hat{i} + \frac{\mu_0 K}{4} x \, \hat{j}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi r^2} \oint (\hat{r}\cdot\vec{r}\,')\, d\vec{r}\,'$$

$$\vec{r} \times (\vec{r}\,' \times d\vec{r}\,') = \vec{r}\,'(\vec{r}\cdot d\vec{r}\,') - d\vec{r}\,'(\vec{r}\cdot\vec{r}\,')$$

$$d[\vec{r}\,'(\vec{r}\cdot\vec{r}\,')] = d\vec{r}\,'(\vec{r}\cdot\vec{r}\,') + \vec{r}\,'(\vec{r}\cdot d\vec{r}\,')$$

$$d\vec{r}\,'(\vec{r}\cdot\vec{r}\,') = -\frac{1}{2}\vec{r} \times (\vec{r}\,' \times d\vec{r}\,') + \underline{\underline{\frac{1}{2} d[\vec{r}\,'(\vec{r}\cdot\vec{r}\,')]}}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi r^3}\left[-\frac{1}{2}\vec{r} \times \underline{\underline{\oint (\vec{r}\,' \times d\vec{r}\,')}}\right]$$

Magnetic Dipole moment

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{r}\,'}{|\vec{r} - \vec{r}\,'|}$$

$$= \frac{\mu_0 I}{4\pi} \oint \sum_{\ell=0}^{\infty} \frac{1}{r^{\ell+1}} r'^{\ell} \cdot P_\ell(\cos\theta)\, d\vec{r}\,'$$

$$= \underbrace{\frac{\mu_0 I}{4\pi r} \oint d\vec{r}\,'}_{=\,0} + \frac{\mu_0 I}{4\pi r^2} \oint r' \cos\theta \, d\vec{r}\,'$$

$$= \frac{\mu_0 I}{4\pi r^2} \oint (\hat{r} \cdot \vec{r}\,')\, d\vec{r}\,'$$

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3}}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \vec{\nabla} \times \left(\vec{m} \times \frac{\vec{r}}{r^3} \right)$$

$$= \frac{\mu_0}{4\pi} \left[\vec{m} \left(\nabla \cdot \frac{\vec{r}}{r^3} \right) - (\vec{m} \cdot \vec{\nabla}) \frac{\vec{r}}{r^3} \right]$$

$$= \frac{\mu_0}{4\pi} \left[\frac{3(\vec{m} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3} \right]$$

$$\oint \vec{B} \cdot \vec{ds} = 0$$

$$\boxed{B_{2n} = B_{1n}}$$

$$\nabla \cdot A = 0 \Rightarrow \boxed{A_{2n} = A_{1n}}$$