Dipan Ghosh
(29/9/11)

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Lorentz Force

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right) I \, \frac{\vec{dl} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

Permeability of free space

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} d^3r'$$

$$= \frac{\mu_0}{4\pi} \int \left[\nabla\left(\frac{1}{|\vec{r} - r'|}\right) \times \vec{J}(\vec{r}\,')\right] d^3r'$$

$$= \frac{\mu_0}{4\pi} \int \left[\nabla \times \left(\frac{1}{|\vec{r} - r'|} J(\vec{r}\,')\right)\right] d^3r'$$

$$= \frac{\mu_0}{4\pi} \nabla \times \int \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d^3r'.$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \nabla(\nabla \cdot A) - \nabla^2 A$$

$$\nabla \times \vec{B} = \frac{\mu_0}{4\pi} \nabla \times \left(\nabla \times \int \frac{\vec{J}(r')}{|\vec{r} - \vec{r}'|} d^3r' \right)$$

$$= \frac{\mu_0}{4\pi} \left[\nabla \left(\nabla \cdot \int \frac{\vec{J}(r')}{|\vec{r} - r'|} d^3r' \right) - \nabla^2 \int \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r' \right].$$

$$\nabla \times \vec{B} = -\frac{\mu_0}{4\pi} \nabla^2 \int \frac{\vec{J}(r')}{|\vec{r}-\vec{r}'|} d^3r'$$

$$= -\frac{\mu_0}{4\pi} \int \vec{J}(\vec{r}') \underbrace{\nabla^2 \frac{1}{|\vec{r}-\vec{r}'|}}_{\Downarrow \; -4\pi\, \delta^3(\vec{r}-\vec{r}')} d^3r'$$

$$= \mu_0 \vec{J}(\vec{r})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\int (\vec{\nabla} \times \vec{B}) \cdot \hat{n} \, ds = \mu_0 \int \vec{J} \cdot \hat{n} \, ds$$

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I}$$

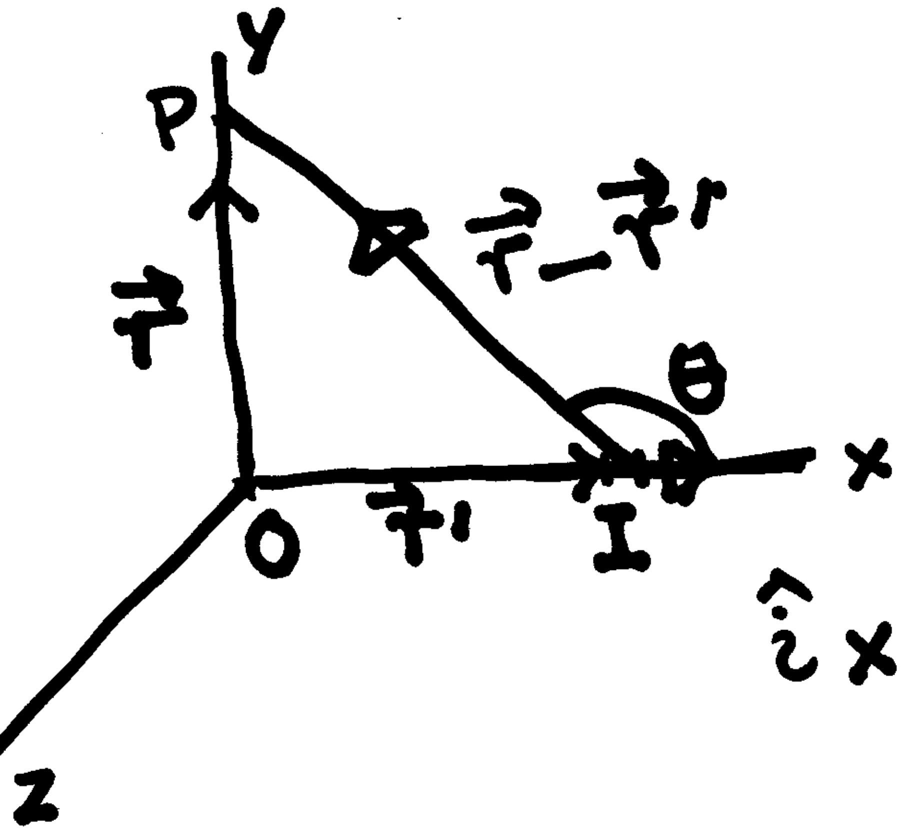
Ampere's Law.

$$\boxed{\oint \vec{B} \cdot d\vec{S} = 0}$$

Gauss's Law.

$$\oint \vec{B} \cdot d\vec{l} = |B| \cdot 2\pi r = \mu_0 I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\varphi}$$



$$\hat{i} \times (\vec{r}-\vec{r}') = \hat{k}\,|\vec{r}-\vec{r}'|\,\sin\theta$$

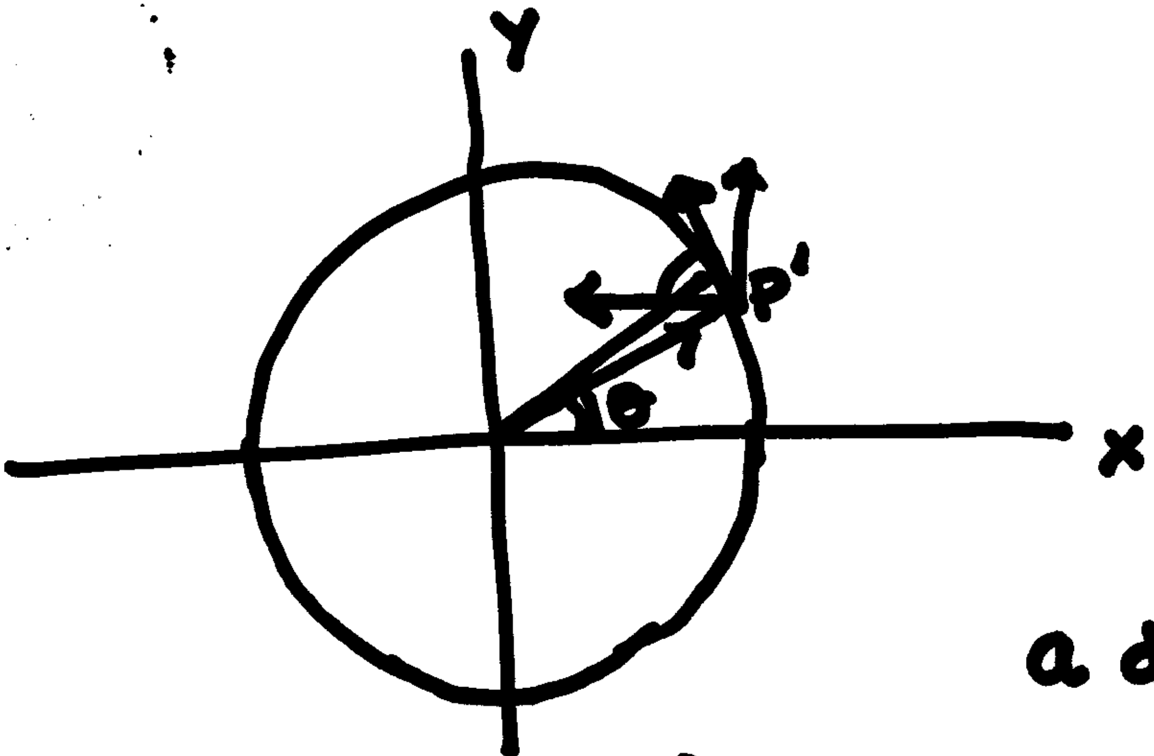
$$x = r\tan\left(\theta - \frac{\pi}{2}\right)$$

$$= -r\cot\theta.$$

$$dx = r\,\mathrm{cosec}^2\theta.\,d\theta.$$

$$|\vec{r}-\vec{r}'| = r\,\mathrm{cosec}\,\theta.$$

$$\vec{B} = \hat{k}\,\frac{\mu_0 I}{4\pi}\int_0^{\pi}\frac{\sin\theta}{r^2\mathrm{cosec}^2\theta}\,r\,\mathrm{cosec}^2\theta.\,d\theta$$



$a\,d\theta$

$$\vec{dl} = -a\,d\theta \sin\theta \cdot \hat{i} + a\,d\theta \cos\theta\,\hat{j}$$

$$\vec{r}\,' = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}.$$