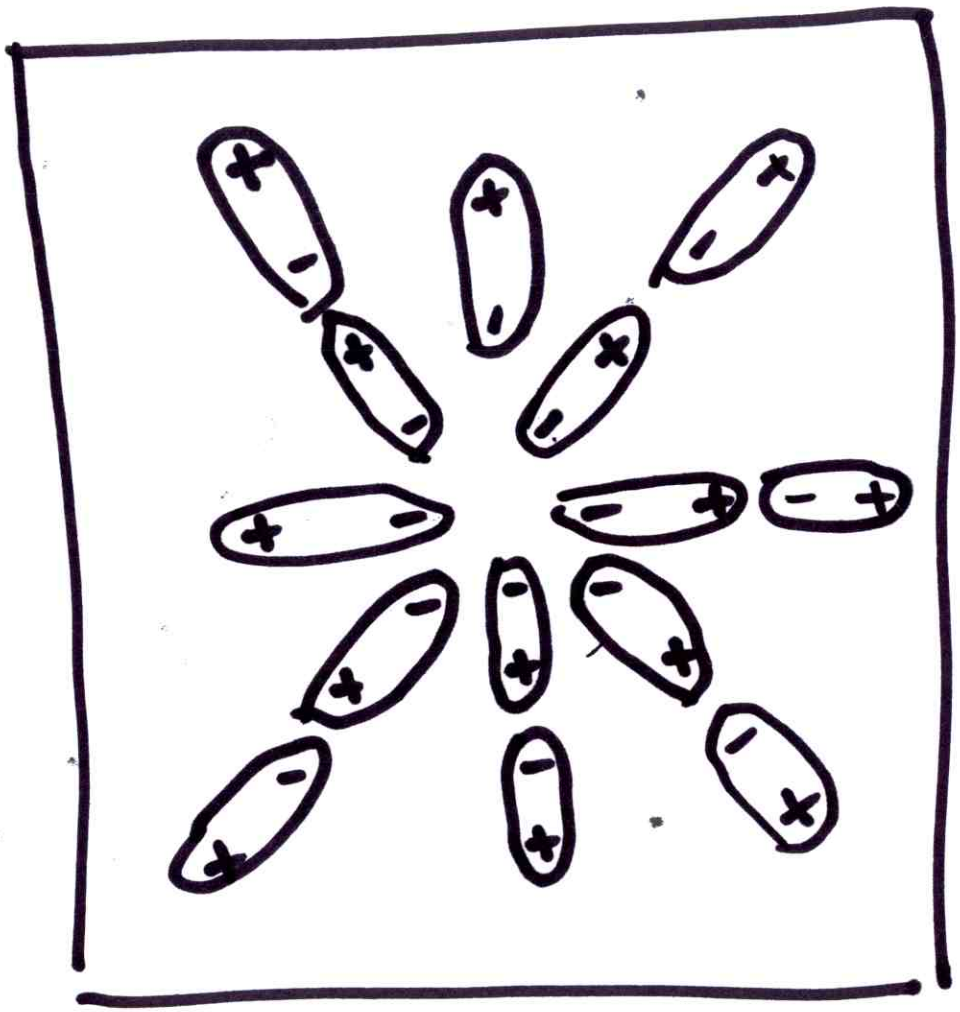
Prof. D. Ghosh
Lec-21
Date. 23-3-2011



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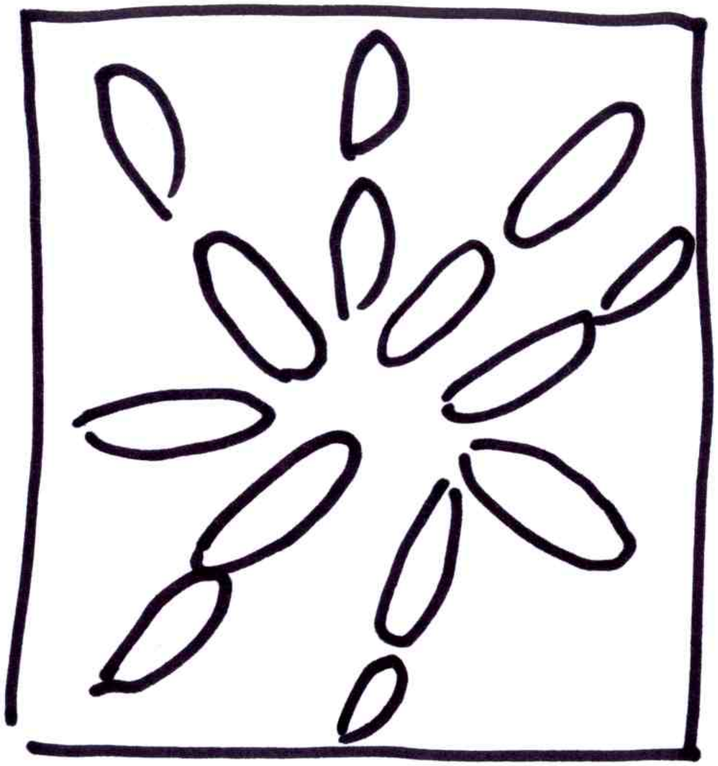
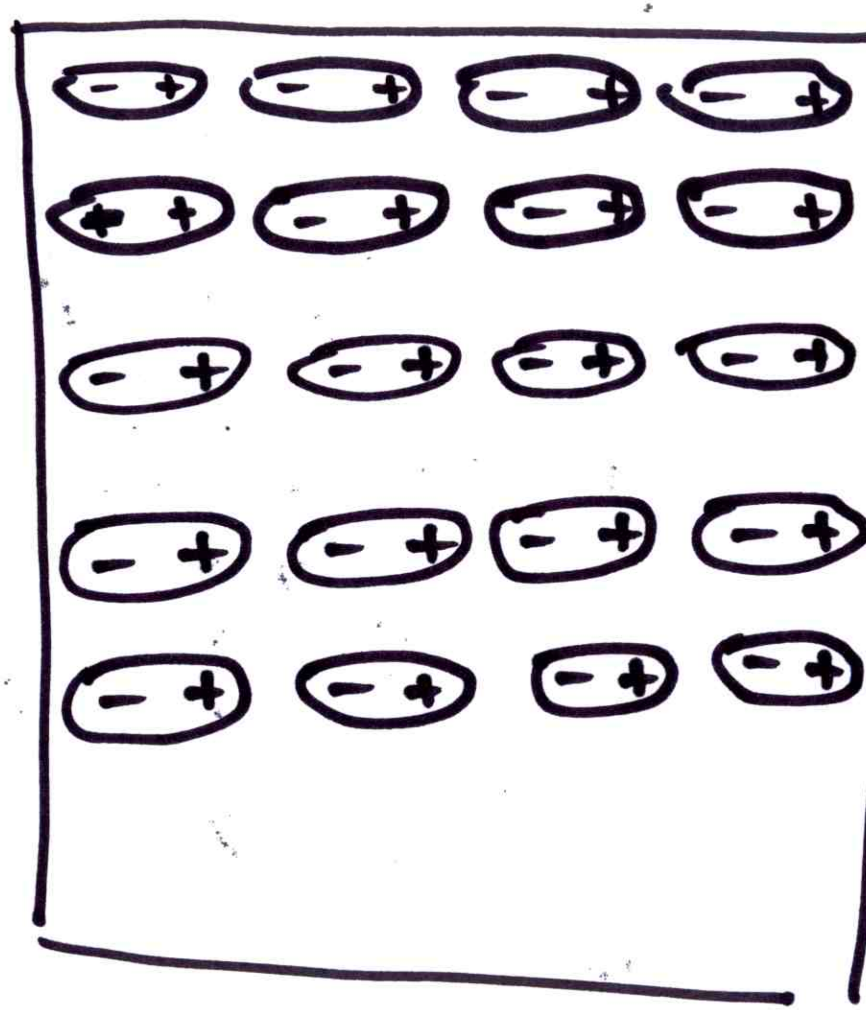
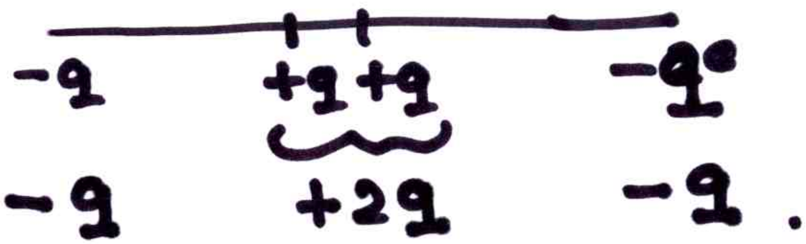
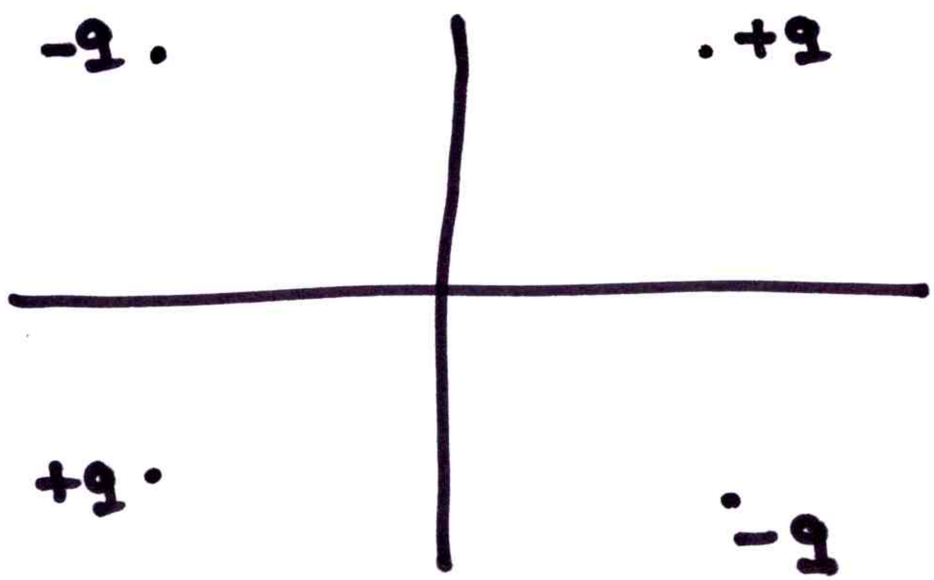




$$\underbrace{\int \vec{r}\,' \, \rho(r') \, d^3r'}_{\vec{p}}.$$



Linear Quadrupole



$$\vec{p} = \int \rho(\vec{r})\,\vec{r}\,d^3r.$$

$$\vec{r}\,' = \vec{r} + \vec{a}.$$

$$\vec{p} = \int \rho(\vec{r}\,')\,(\vec{r}\,' - \vec{a})\,d^3r'.$$

$$= \int \rho(\vec{r}\,')\,\vec{r}\,'\,d^3r' - \vec{a}\underbrace{\int \rho(r')\,d^3r'}_{Q}$$

If $Q = 0$

$\vec{p}$ is independent of origin.

$$\boxed{\begin{aligned} \rho_b &= -\vec{\nabla}\cdot\vec{P} \\ \sigma_b &= \vec{P}\cdot\hat{n} \end{aligned}}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_f + \rho_b}{\epsilon_0}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P}$$

$$\vec{\nabla}\cdot\vec{D} = \epsilon_0\vec{\nabla}\cdot\vec{E} + \nabla\cdot\vec{P}$$

$$= \epsilon_0 \cdot \frac{\rho_f + \rho_b}{\epsilon_0} - \rho_b = \rho_f$$

$$E = E_0 \hat{z}$$

$$\phi_{ext} = -E_0 r \cos\theta \quad \text{(At Large distances)}.$$

$$\phi_1(r,\theta) = A_1 r \cos\theta + \frac{B_1}{r^2}\cos\theta .$$

$$\phi_2(r,\theta) = A_2 r \cos\theta$$

$$A_1 = -E_0$$

$$-E_0 a + \frac{B_1}{a^2} = A_2 a$$

$$D_{1n} = \frac{1}{4\pi}\left(-\frac{\partial \Phi_1}{\partial n}\right)$$

$$= \frac{1}{4\pi}\left[-\frac{\partial}{\partial z}\left[\qquad\qquad\qquad\qquad\qquad\right]\right]$$

$$= \frac{-\cancel{1}}{4\pi}\cdot\left[\frac{\left(-\frac{1}{2}\right)\ 2(z-d)q}{[(z-d)^2+\rho^2]^{3/2}} + \frac{2(z+d)q'\left(-\frac{1}{2}\right)}{[(z+d)^2+\rho^2]^{3/2}}\right].$$

$$\int \vec{\nabla} \cdot \vec{D} \, d^3r = \int \rho_f \, d^3r$$

$$\boxed{\oint \vec{D} \cdot \vec{ds} = Q_{free}}$$