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$$\Phi = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\sqrt{\rho^2+(z-d)^2}} - \frac{1}{\sqrt{\rho^2+(z+d)^2}}\right].$$

$$\vec{E} = -\nabla\Phi = \left(-\hat{\rho}\frac{\partial}{\partial\rho} - \hat{k}\frac{\partial}{\partial z}\right)\Phi$$

$$= \frac{q}{4\pi\epsilon_0}\left\{\left[\hat{\rho}\,\frac{\rho}{[\rho^2+(z-d)^2]^{3/2}} - \hat{\rho}\,\frac{\rho}{[\rho^2+(z+d)^2]^{3/2}}\right], + \hat{k}\,\underbrace{\left(\frac{z-d}{[\rho^2+(z-d)^2]^{3/2}} - \frac{z+d}{[\rho^2+(z+d)^2]^{3/2}}\right)}\right\}.$$

$$\sigma = \epsilon_0 E_z = -\frac{q\,d}{2\pi(\rho^2+d^2)^{3/2}}$$

$$Q_{induced} = \int \sigma \, . \, dx\, dy$$

$$= -\frac{qd}{2\pi} \int_0^\infty \frac{2\pi \rho \, d\rho}{(\rho^2+d^2)^{3/2}}.$$

$$= \left. \frac{qd}{(\rho^2+d^2)^{1/2}} \right|_0^\infty = -q \, .$$

$$\vec{E}(0, d) = -\hat{k}\,\frac{q}{4\pi\epsilon_0}\,\frac{1}{4d^2}.$$

Force exerted on the surface

$$\vec{F} = \int \sigma E_z \, d^2\rho$$

$$= \frac{q^2 d^2}{8\pi^2\epsilon_0}\int_0^\infty \frac{2\pi\rho\, d\rho}{(\rho^2+d^2)^3} = \frac{q^2}{16\pi\epsilon_0 d^2}.$$

$$E_z = -\frac{\partial \phi}{\partial y}\bigg|_{y=0} =$$

$$\phi(r,\theta) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_1} + \frac{q'}{r_2}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{a^2+r^2-2ar\cos\theta}} + \frac{q'}{\sqrt{b^2+r^2-2br\cos\theta}}\right].$$

$$r = R \qquad \Phi(r,\theta) = 0$$

$$q^2(\underbrace{b^2+r^2}-\underline{\underline{2br\cos\theta}}) = q'^2(\underbrace{a^2+r^2}-\underline{\underline{2ar\cos\theta}})$$

$$\frac{q^2}{q'^2} = \frac{a}{b} \quad \therefore\ q' = -\sqrt{\frac{b}{a}}\cdot q.$$

$$b^2+r^2 = \frac{q'^2}{q^2}(a^2+r^2) = \frac{b}{a}(a^2+r^2)\Big|_{r=R}$$

$$\boxed{ab = R^2}$$

$$q' = -\sqrt{\frac{b}{a}} \cdot q$$

$$= -\sqrt{\frac{R^2}{a^2}} \cdot q = -\frac{R}{a} q .$$

$$b = \frac{R^2}{a}$$