$\frac{\partial E_z}{\partial x}$; $\frac{\partial E_z}{\partial y}$ must be finite

$$\nabla \times E = 0$$

$$(\nabla \times E)_x = \underline{\underline{\frac{\partial}{\partial y} E_z}} - \underline{\underline{\frac{\partial}{\partial z} . E_y}} = 0 .$$

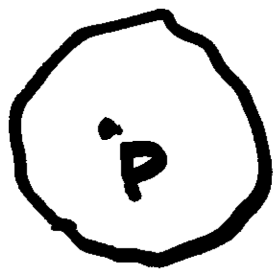
$\downarrow$

finite and continuous.

$\Downarrow$

$\frac{\partial}{\partial z} E_x$

$$E_n = -\frac{\partial \varphi}{\partial n} = \frac{\sigma}{\epsilon_0} \Rightarrow -\epsilon_0 \oint \frac{\partial \varphi}{\partial n}\, ds$$



$$\frac{\partial \varphi}{\partial n} < 0$$

$$\oint \frac{\partial \varphi}{\partial n} \cdot ds < 0 \Rightarrow$$ Contradicts original assumption.

$$W = \frac{\epsilon_0}{2}\int_{V'} |E|^2 \, dV$$

$$= -\frac{\epsilon_0}{2}\int_{V'} \vec{E}\cdot\nabla\varphi \, dV.$$

$$= -\frac{\epsilon_0}{2}\int_{V'} \nabla\cdot(\varphi\vec{E})\, dV + \underbrace{\frac{\epsilon_0}{2}\int \varphi\,\vec{\nabla}\cdot\vec{E}\, dV}$$

$$\nabla\cdot(\varphi\vec{E}) = \overrightarrow{\nabla\varphi}\cdot\vec{E} + \varphi\,\vec{\nabla}\cdot\vec{E}$$

$$W = -\frac{\epsilon_0}{2}\int_{V'} \nabla\cdot(\varphi\vec{E})\, dV = \frac{\epsilon_0}{2}\int_{\substack{\text{Surface of}\\ \text{a conductor}}} \varphi\,\hat{n}\cdot\vec{E}\, ds.$$

$$Q_i = \sum_j C_{ij} \varphi_j$$

$$\varphi_i = \sum_j P_{ij} Q_j$$

For a single conductor

$$Q = C\varphi$$

P_{ij} : Coefficients of potential

C_{ij} : Coefficients of capacitance

$$W = \frac{1}{2}\sum_{i,j} p_{ij} Q_i Q_j = \frac{1}{2}\sum_{ij} C_{ij}\,\varphi_i\varphi_j$$

Add ΔQ_k to k-th conductor.

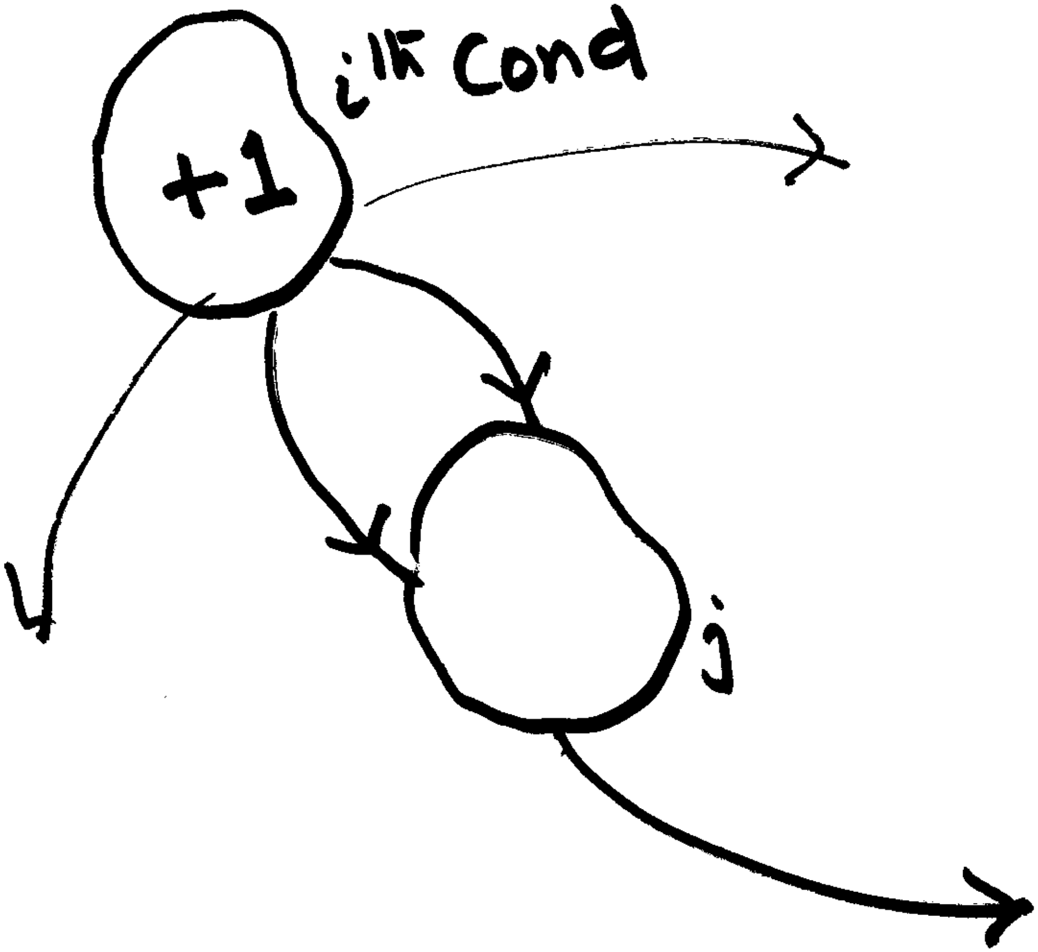
$$\Delta W = \frac{\partial W}{\partial Q_k}\cdot\Delta Q_k.$$

$$= \frac{1}{2}\sum_j (p_{jk} + p_{kj})\, Q_j\,\Delta Q_k.$$

$$\equiv \varphi_k\cdot\Delta Q_k. = \sum_j p_{kj}\, Q_j\,\Delta Q_k$$

$$\boxed{p_{jk} = p_{kj}}$$

5.



$p_{ii} > 0$

$p_{ii} > p_{ij} > 0$

$$\int_V (\varphi \nabla^2 \psi - \psi \nabla^2 \varphi)\, dV = \oint_S (\varphi \nabla \psi - \psi \nabla \varphi) \cdot \hat{n}\, ds.$$

Green's Theorem.

$$\nabla^2 \varphi = -\frac{\rho}{\epsilon_0}$$

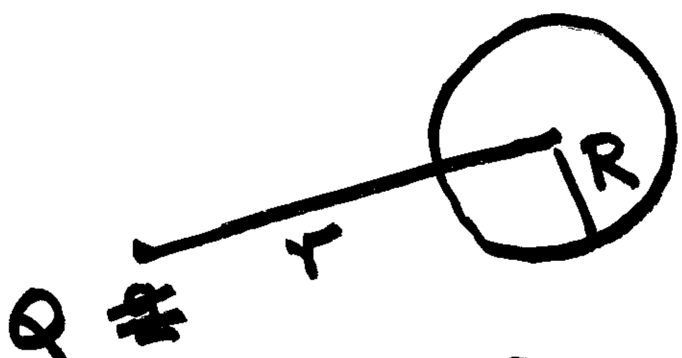
$$\nabla^2 \psi = -\frac{\rho'}{\epsilon_0}$$

$$-\frac{1}{\epsilon_0} \int_V (\varphi \rho' - \psi \rho)\, dV = -\oint (\varphi \vec{E}' - \psi \vec{E}) \cdot \hat{n}\, ds$$

$$= \frac{1}{\epsilon_0} \oint_S (\varphi \sigma' - \psi \sigma)\, ds.$$

$$\int_V \varphi \rho' \, dV + \oint \varphi \sigma' \, ds = \int_V \psi \rho \, dV + \oint \psi \sigma \, ds$$

Green's Reciprocity Theorem.



$r > R$.

$$\varphi = \frac{Q}{4\pi\epsilon_0 r}.$$

$$Q \cdot \frac{Q}{4\pi\epsilon_0 r} = Q \cdot \psi$$

$$\boxed{\psi = \frac{Q}{4\pi\epsilon_0 r}}$$

1 : Point charge

2 : Sphere

$$\varphi_1 = p_{11} q_1 + p_{12} q_2$$
$$\varphi_2 = p_{21} q_1 + p_{22} q_2$$

Charge on sphere $q_2 = q$; $q_1 = 0$

$$\varphi_1 = p_{12} q . \quad \therefore p_{12} = \frac{1}{4\pi\epsilon_0 r} = p_{21}$$

$$\varphi_2 = p_{21} q_1 + p_{22} q_2$$
$$= \underline{\underline{p_{21}}}\, q = \frac{1}{4\pi\epsilon_0 r} \cdot q .$$

$q_1 \qquad q_2 = 0$

$$\varphi_1 = \frac{Q_1}{C_1} \; ; \quad \varphi_2 = \frac{Q_1}{4\pi\epsilon_0 r}$$

$$\varphi_i = \sum_j p_{ij} Q_j$$

$$P = \begin{pmatrix} \frac{1}{C_1} & \frac{1}{4\pi\epsilon_0 r} \\ \frac{1}{4\pi\epsilon_0 r} & \frac{1}{C_2} \end{pmatrix}$$

$$P^{-1} =$$

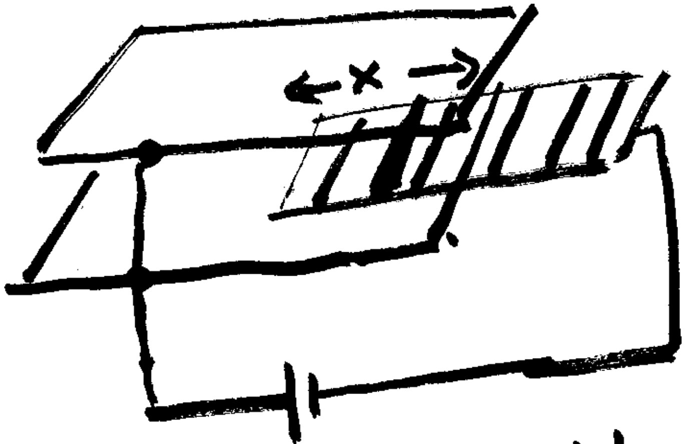
$$\vec{F} \cdot d\vec{r} = -dW$$

$$F_x = -\frac{\partial W}{\partial x} \Rightarrow \vec{F} = -\nabla W.$$

$$W = \frac{1}{2} \sum_i \varphi_i q_i$$

$$W^{es} = \frac{Q^2}{2C} = \frac{Q^2 x}{2A\epsilon_0}$$

$$F_x = -\frac{dW}{dx} = -\frac{Q^2}{2A\epsilon_0}$$



$$|E| = \frac{V}{d/2} = \frac{2V}{d}.$$

$$W^{es} = \frac{\epsilon_0}{2}\int |E|^2 \, d\tau$$

$$= \frac{\epsilon_0}{2}\left(\frac{2V}{d}\right)^2 x\, d\, h$$

$$F = \frac{2\epsilon_0 V^2 h}{d}.$$