Prof. D. Ghosh - Lec - 9.
Date 3/8/10

$$\vec{\nabla} \times \vec{E} = 0$$

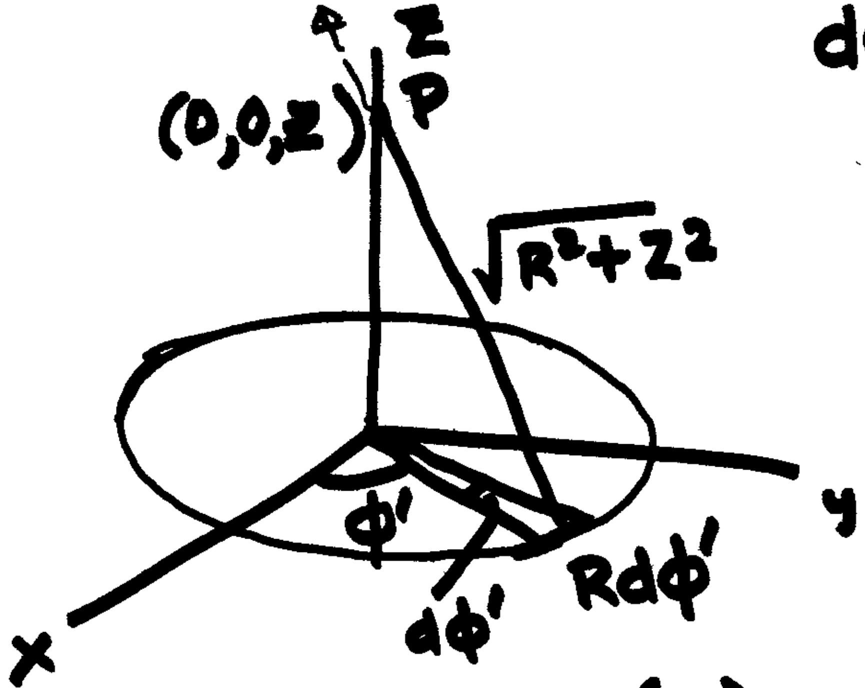
$$\vec{E} = -\vec{\nabla} \varphi$$

$$W = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{F}\cdot d\vec{l} = -q\int_{\vec{r}_A}^{\vec{r}_B} \vec{E}\cdot d\vec{l}$$

$$= -q\int_{\vec{r}_A}^{\vec{r}_B} -(\vec{\nabla}\phi)\cdot d\vec{l}$$

$$= +q\int_{r_A}^{\vec{r}_B} d\phi = q\left[\phi(B) - \phi(A)\right].$$

$$W = q\,\phi(B)$$



$$dq = \lambda R \, d\varphi'$$

$$\varphi(z) = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda R \, d\varphi'}{\sqrt{R^2+z^2}}$$

$$= \frac{\lambda}{2\epsilon_0} \cdot \frac{R}{\sqrt{R^2+z^2}}$$

$$\vec{E} = -\nabla\phi = -\hat{k}\,\frac{\lambda}{2\epsilon_0} R \frac{\partial}{\partial z} \frac{1}{\sqrt{R^2+z^2}} = \hat{k}\,\frac{\lambda R}{2\epsilon_0}\left(\frac{z}{(R^2+z^2)^{3/2}}\right),$$

$$\varphi(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\, ds'}{|\vec{r}-\vec{r}\,'|}$$

$$\varphi(z) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma R^2 \sin\theta'\, d\theta'\, d\varphi'}{\sqrt{R^2+z^2-2Rz\cos\theta'}}$$

$$= \frac{2\pi\sigma R^2}{4\pi\epsilon_0}\int_{-1}^{+1} \frac{d\mu}{\sqrt{R^2+z^2-2Rz\mu}}$$

$\mu = \cos\theta'$

$d\mu = -\sin\theta'\, d\theta'$

$$= \frac{\sigma R^2}{2\epsilon_0}\left(-\frac{1}{Rz}\right)\sqrt{R^2+z^2-2Rz\mu}\,\Bigg|_{-1}^{+1}$$

$$= \frac{\sigma R^2}{2\epsilon_0}\left(-\frac{1}{Rz}\right)\left[\sqrt{(R-z)^2}-\sqrt{(R+z)^2}\right]$$

$R > z$

$$\varphi(z) = \frac{\sigma R^2}{\cancel{2}\epsilon_0}\left(-\frac{1}{Rz}\right)(-\cancel{2}z)$$

$\theta(r-R)$.

$$= \frac{\sigma R^2}{\cancel{2}\epsilon_0 \cdot R} = \frac{Q}{4\pi\epsilon_0 R}$$

Independent of z.

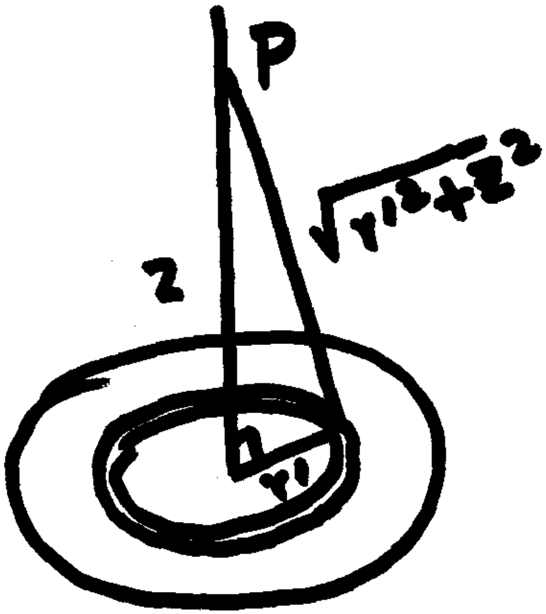
$z > R$

$$\varphi(z) = \frac{Q}{4\pi\epsilon_0 z} \qquad \underline{\underline{z \rightarrow r}}.$$

$$\varphi(r) = \underbrace{\theta(r-R)}\frac{Q}{4\pi\epsilon_0 r} + \theta(R-r)\frac{Q}{4\pi\epsilon_0 R}$$

$$\vec{E}(r) = -\nabla\phi = -\hat{r}\cdot\theta(r-R)\frac{Q}{4\pi\epsilon_0 r^2}$$

$\sigma . 2\pi r' dr'$



$$\phi(z) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma . 2\pi r' dr'}{\sqrt{r'^2+z^2}}.$$

$$= \frac{\sigma}{2\epsilon_0}\left(\sqrt{r'^2+z^2}\right)\Big|_0^R$$

$$= \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2} - |z|\right)$$

$\underline{z \gg R}$
$$= \frac{\sigma}{2\epsilon_0}\left(|z|\sqrt{1+\frac{R^2}{z^2}} - |z|\right)$$

$$= \frac{\sigma}{2\epsilon_0} . \frac{R^2}{2|z|} = \frac{Q}{4\pi\epsilon_0 |z|}$$

$R \gg z$

$$\frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2}-|z|\right)$$

$$= \frac{\sigma R}{2\epsilon_0}.$$

$$\vec{E} = -\nabla\phi = -\hat{k}\,\frac{\sigma}{2\epsilon_0}\,\frac{\partial}{\partial z}\left(\sqrt{R^2+z^2}-|z|\right)$$

$$= -\hat{k}\,.\,\frac{\sigma}{2\epsilon_0}\left(\frac{z}{\sqrt{R^2+z^2}} - \operatorname{sgn}(z)\right)$$

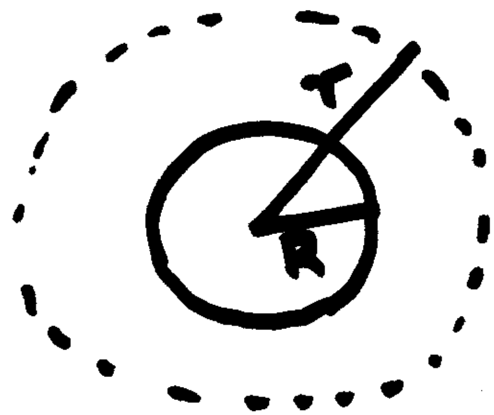
Near the disk.

$$\vec{E} = \hat{k}\,\frac{\sigma}{2\epsilon_0}\,\operatorname{sgn}(z)$$

$$Q = \frac{4\pi}{3} R^3 \cdot \rho$$

$\underline{r > R}$

$$4\pi r^2 \cdot |E| = \frac{Q}{\epsilon_0}$$



$$\vec{E} = \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2} \hat{r}$$

$r < R$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} \cdot r \, \hat{r}$$

$V(\infty) = 0$

$r > R$ $\quad V(r) = \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r}$.

$\underline{r < R}$