Prof. D. Ghosh
Lec NO. 5 13/8/1[illegible]

$\nabla^2\left(\frac{1}{r}\right)$?

$$\vec{\nabla}\cdot\vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V_r\right)$$

$$\nabla^2\left(\frac{1}{r}\right) = \vec{\nabla}\cdot\nabla\left(\frac{1}{r}\right)$$

$$= \vec{\nabla}\cdot\frac{\vec{r}}{r^3}$$

$$= \frac{\vec{\nabla}\cdot\vec{r}}{r^3} + \vec{r}\cdot\nabla\left(\frac{1}{r^3}\right)$$

$$= \frac{3}{r^3} - \frac{3}{r^3} = 0$$

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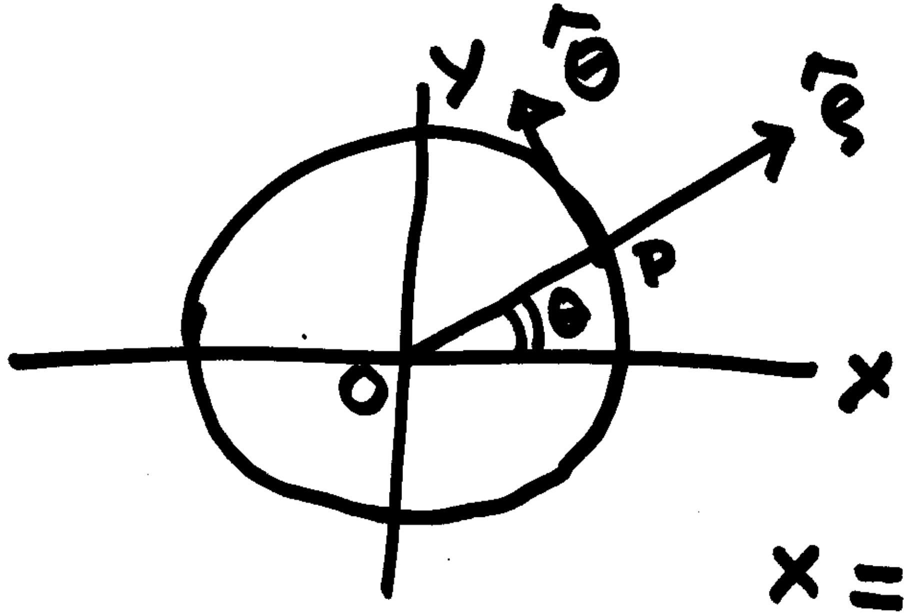

$$x = \varrho \cos\theta$$
$$y = \varrho \sin\theta$$

$$\int_{-\infty}^{+\infty} e^{ikx}\,dk = \lim_{n\to\infty} \int_{-n}^{+n} e^{ikx}\,dk$$

$$= \lim_{n\to\infty} \left.\frac{e^{ikx}}{ix}\right|_{-n}^{+n}$$

$$= \lim_{n\to\infty} \frac{2\sin nx}{x} = 2\pi\,\delta(x)$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx}\,dk = \delta(x)$$

$$\delta(x) = 0 \quad \text{if} \quad x \neq 0$$

$$\int_{-\epsilon}^{+\epsilon} f(x)\,\delta(\underline{\underline{x}})\,dx = f(x=0)$$

$$\int (\phi \nabla^2 \psi + \nabla\phi \cdot \nabla\psi)\, dV = \oint_S \phi (\nabla\psi)\cdot\hat{n}\, ds$$

$$\phi = \psi$$

$$\int (\phi \underset{0}{\nabla^2 \phi} + |\nabla\phi|^2)\, dV = \underset{=0}{\oint \phi (\vec{\nabla}\vec{\phi}\cdot\hat{n})\, ds}$$

$$\int |\nabla\phi|^2 dV = 0 \Rightarrow$$

$$\nabla\phi = 0 \qquad \vec{C} = 0$$

$$\int (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dv$$

$$= \oint_S (\phi \nabla \psi - \psi \nabla \phi) \cdot \hat{n}\, ds$$

$$\vec{C} = \vec{A} - \vec{B} \quad : \nabla \times \vec{C} = 0$$

$$C = -\nabla \phi$$

$$\vec{\nabla} \cdot \vec{C} = 0 \quad : \boxed{\nabla^2 \phi = 0}$$

$$\vec{C} \cdot \hat{n}\, ds = \nabla \phi \cdot \hat{n}\, ds = 0$$

$$\int_V (\vec{\nabla} \cdot \vec{A})\, dV = \int_S \vec{A} \cdot \hat{n} \, . \, dS$$

$$\vec{A} = \varphi \nabla \psi$$

$$\int_V (\varphi \nabla^2 \psi + \nabla \varphi \cdot \nabla \psi)\, dV$$

$$= \int_S \varphi (\nabla \psi \cdot \hat{n})\, dS$$

$$\int_V (\psi \nabla^2 \varphi + \nabla \phi \cdot \nabla \psi)\, dV$$

$$= \int_S \psi (\nabla \varphi . \hat{n})\, dS$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}.$$

$$\nabla^2 \vec{f}$$

$$\hat{i}\, \nabla^2 f_x + \hat{j}\, \nabla^2 f_y + \hat{k}\, \nabla^2 f_z .$$

$$\vec{\nabla}\cdot(\vec{\nabla} f) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}\right)$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}.$$

$$\nabla^2 f = \left(\underbrace{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}\right) f$$

$$\boxed{\nabla^2 f = 0}$$ <u>Laplace's Eqn.</u>

$$\nabla^2 \vec{f} \ ?$$