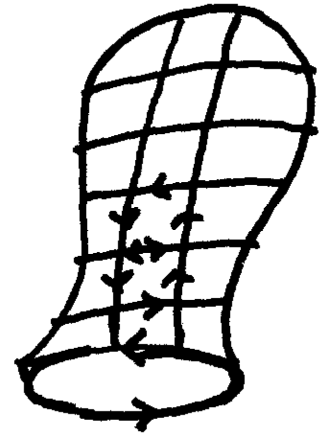
Prof. D. Ghosh
Lec No. 4 . 11/8/

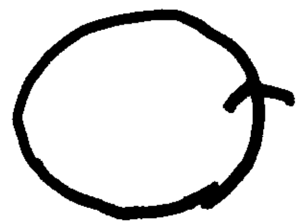
$$\oint \vec{F}\cdot\vec{dl} = \sum_i \boxed{\frac{\oint_{C_i} \vec{F}\cdot\vec{dl}}{\Delta S_i}}\,\Delta S_i$$

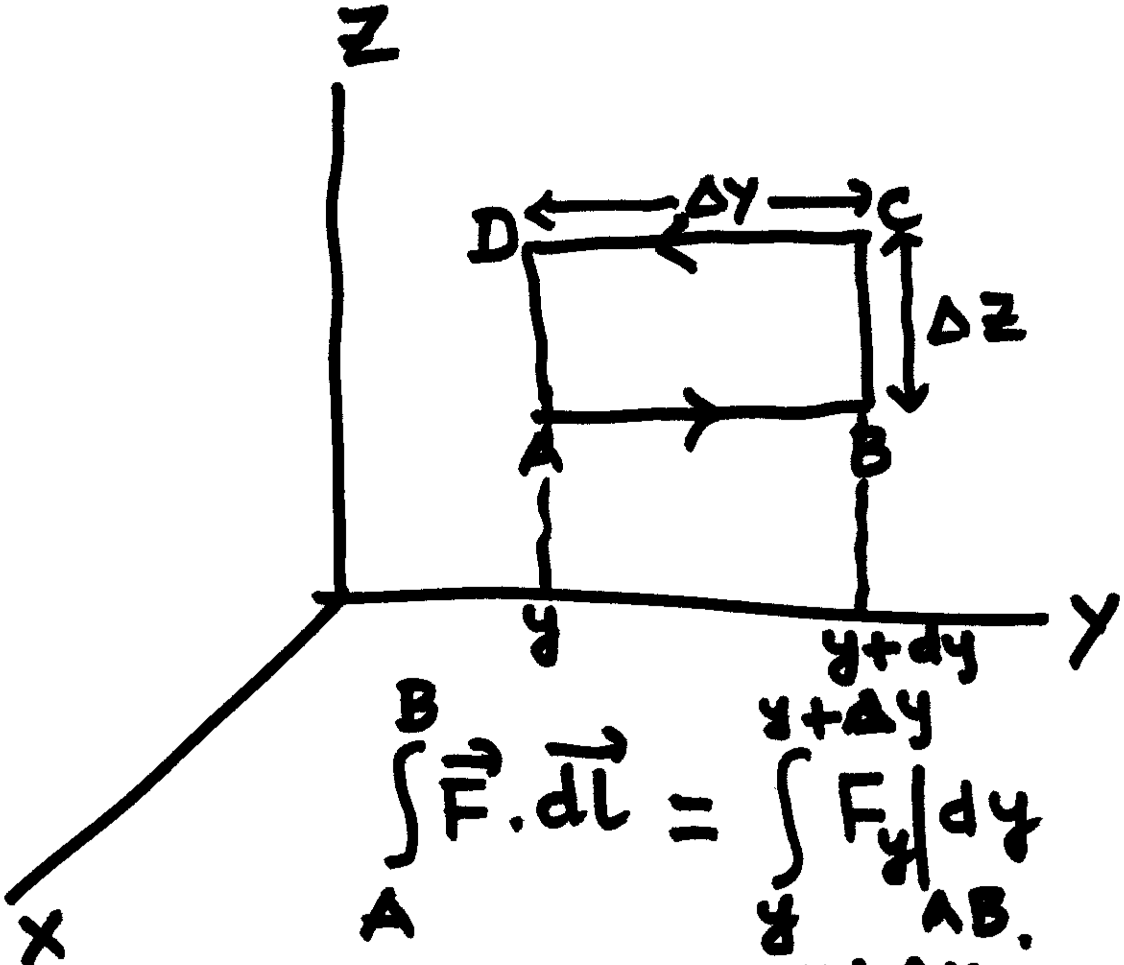


$$\text{Curl}\,\vec{F} = \lim_{\Delta S_i \to 0} \frac{\oint_{C_i} \vec{F}\cdot\vec{dl}}{\Delta S_i}\,\hat{n}_i$$

$$\oint \vec{F}\cdot\vec{dl} = \int_S \text{Curl}\,\vec{F}\cdot\hat{n}\,ds$$

Stoke's Theorem





$$\int_A^B \vec{F}\cdot\vec{dl} = \int_y^{y+\Delta y} F_y\Big|_{AB}\, dy.$$

$$\int_C^D \vec{F}\cdot\vec{dl} = -\int_y^{y+\Delta y} F_y\Big|_{CD}\, dy.$$

$$F_y\Big|_{CD} = F_y\Big|_{AB} + \frac{\partial F_y}{\partial z}\cdot\Delta z$$

AB + CD

$$\int \vec{F}\cdot dl = -\frac{\partial F_y}{\partial z}\cdot\Delta z\ \Delta y$$

BC + DA

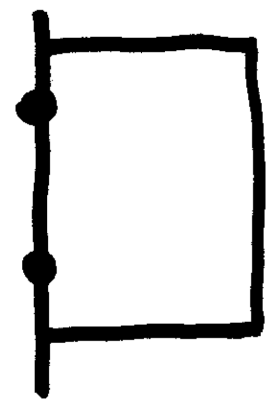
$$= \frac{\partial F_z}{\partial y}\cdot\Delta y\cdot\Delta z.$$

$$\left(\text{Curl}\ \vec{F}\right)_x = \frac{\partial F_z}{\partial y} - \frac{\partial}{\partial z}F_y$$

$$(\vec{\nabla} \times \vec{F})_y = \frac{\partial}{\partial z} F_x - \frac{\partial}{\partial x} F_z$$

$$(\nabla \times \vec{F})_z = \frac{\partial}{\partial x} F_y - \frac{\partial}{\partial y} F_x$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$



$$\vec{F} = -\hat{\imath}\, y + \hat{\jmath}\, x.$$

$$\begin{aligned}\vec{\nabla} \times \vec{F} = {} & \hat{\imath}\left(\cancel{\frac{\partial}{\partial y} F_z} - \cancel{\frac{\partial}{\partial z} F_y}\right) \\ & + \hat{\jmath}\left(\cancel{\frac{\partial}{\partial z} F_x} - \cancel{\frac{\partial}{\partial x} F_z}\right) \\ & + \hat{k}\left(\frac{\partial}{\partial x} F_y - \frac{\partial}{\partial y} F_x\right)\end{aligned}$$

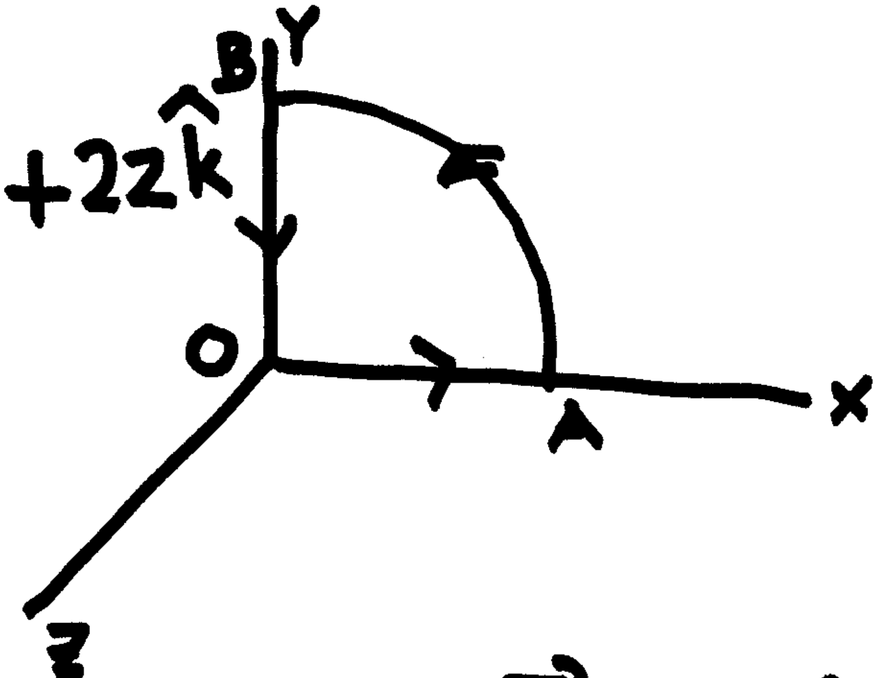
$$= \hat{k}(1+1)$$

$$= 2\hat{k}$$

$\nabla \phi$

$$\boxed{\nabla \times (\nabla \phi) = 0}$$

$$\vec{F} = 4y\hat{i} + x\hat{j} + 2z\hat{k}$$

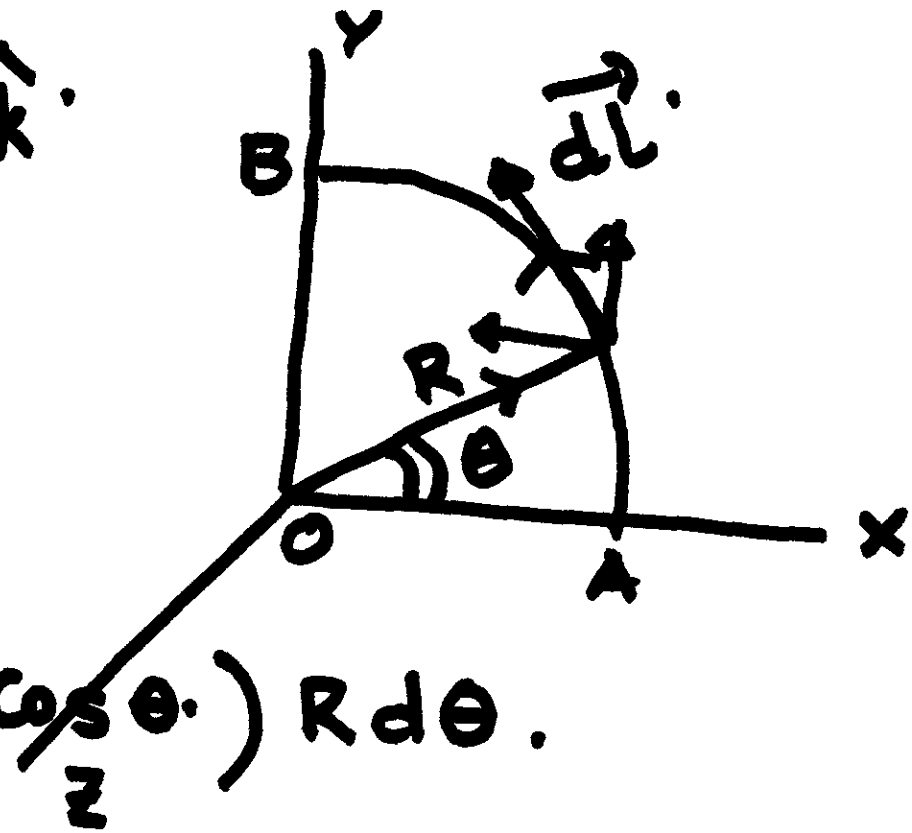


OA : $y = 0,\ z = 0 \quad \vec{F} = x\hat{j}$.

$$d\vec{l} = \hat{i}\,dx$$

$$\int_O^A \vec{F}\cdot d\vec{l} = 0$$

$$\vec{F} = 4y\,\hat{i} + x\,\hat{j} + 2z\,\hat{k}.$$



$$x = R\cos\theta$$
$$y = R\sin\theta$$

$$\vec{dl} = (-\hat{i}\sin\theta + j\cos\theta.)\,R\,d\theta.$$

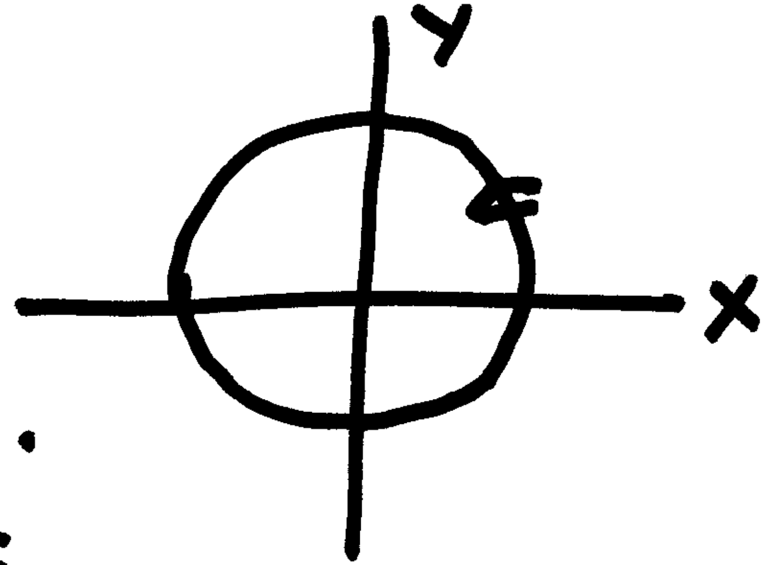
$$\int_0^{\pi/2} \vec{F}\cdot\vec{dl} = \int_0^{\pi/2} (-4\sin^2\theta + \cos^2\theta)\,d\theta.$$
$$= -4\times\frac{\pi}{4} + \frac{\pi}{4} = -\frac{3\pi}{4}$$

$$\vec{\nabla} \times \vec{F} = -3\hat{k}.$$

$$\int (\vec{\nabla} \times \vec{F}) \cdot \hat{n} \, ds$$

$$\int -3 \, ds = -\frac{3}{4}\pi R^2.$$

$$\vec{F} = -\hat{i}\,y + \hat{j}\,z + \hat{k}\,x^2.$$

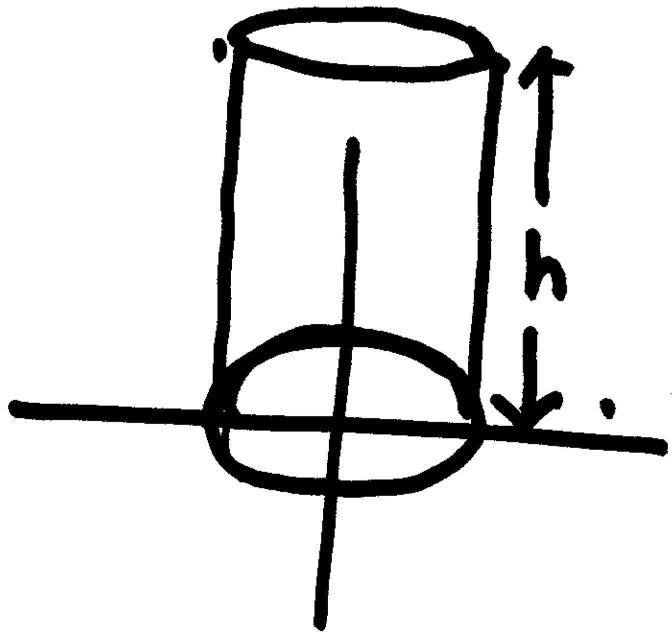


$$\vec{\nabla}\times\vec{F}$$

$$= -\hat{i} + 2\hat{j}\,x + \underline{\underline{\hat{k}}}.$$

$$\int (\vec{\nabla}\times\vec{F})\cdot\hat{n}\,ds$$

$$= \int ds = \pi R^2.$$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