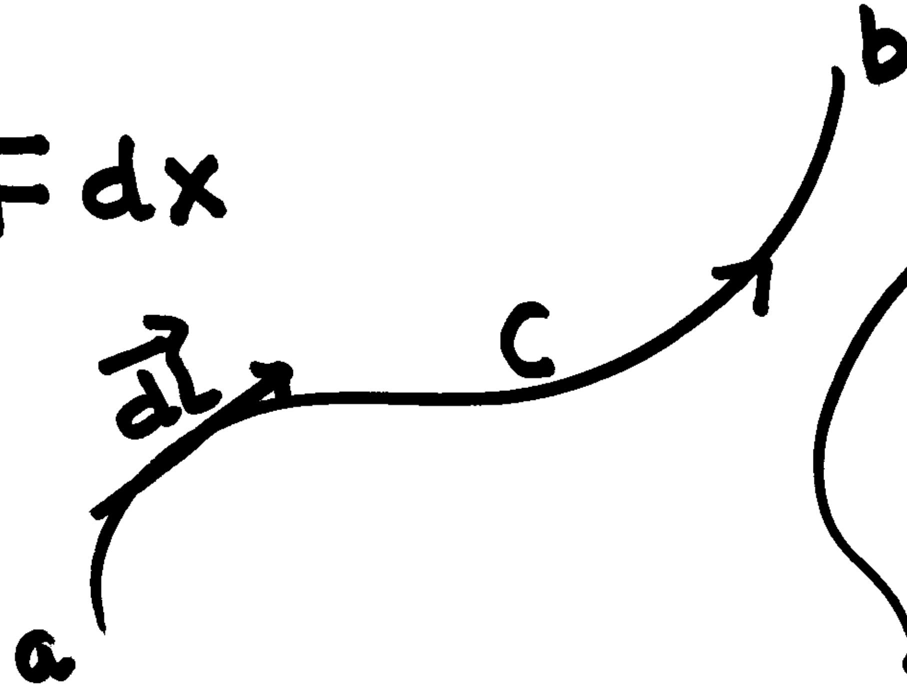
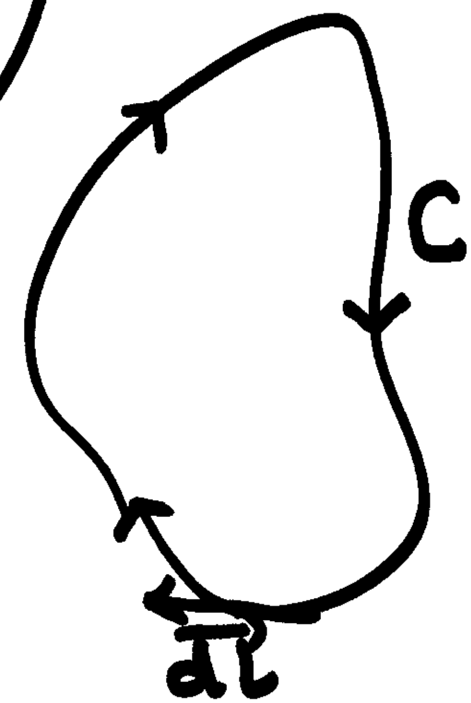
Prof. D. Ghosh.
Lec-2
Date-30-7-10

$$\int_a^b F\,dx$$





$$\int \vec{F}\cdot\vec{dl}$$

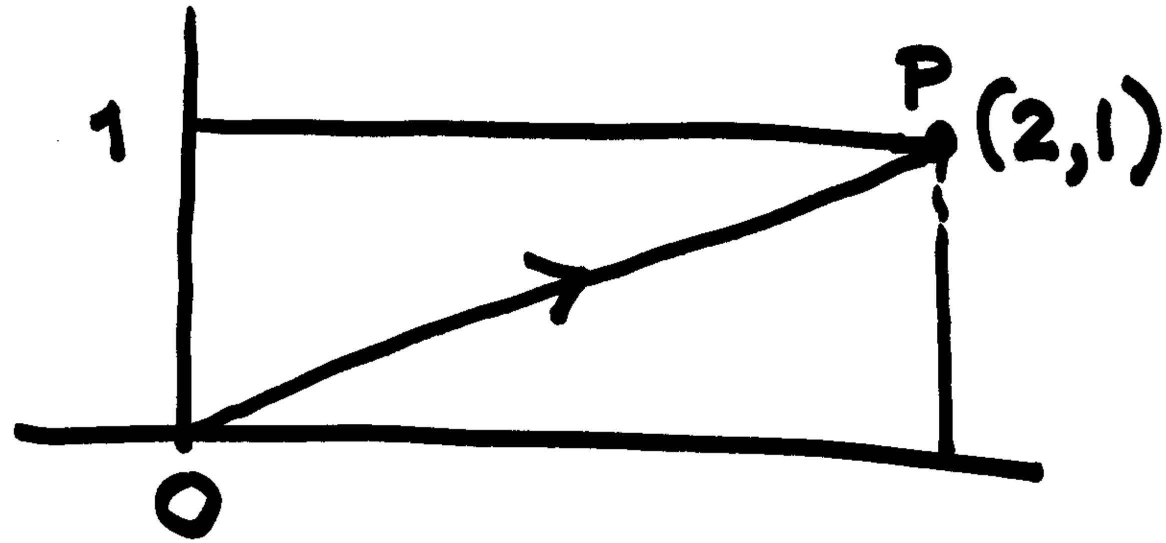
$$\int_C \vec{F} \cdot \overrightarrow{dr} = \int_C \vec{F} \cdot \frac{\overrightarrow{dr}}{dt}\, dt$$

$$= \int_C \vec{F} \cdot \vec{v}\, dt$$

$$\vec{F} = (x^2 - y^2)\,\hat{\imath} + 2xy\,\hat{\jmath}$$

$x = 2y$

$y = \frac{x}{2}$



$$\int \vec{F}\cdot\vec{dl} = \int F_x\,dx + \int F_y\,dy$$

$$= \int_0^2 \left(x^2 - \frac{x^2}{4}\right)dx + \int_0^1 4y^2\,dy$$

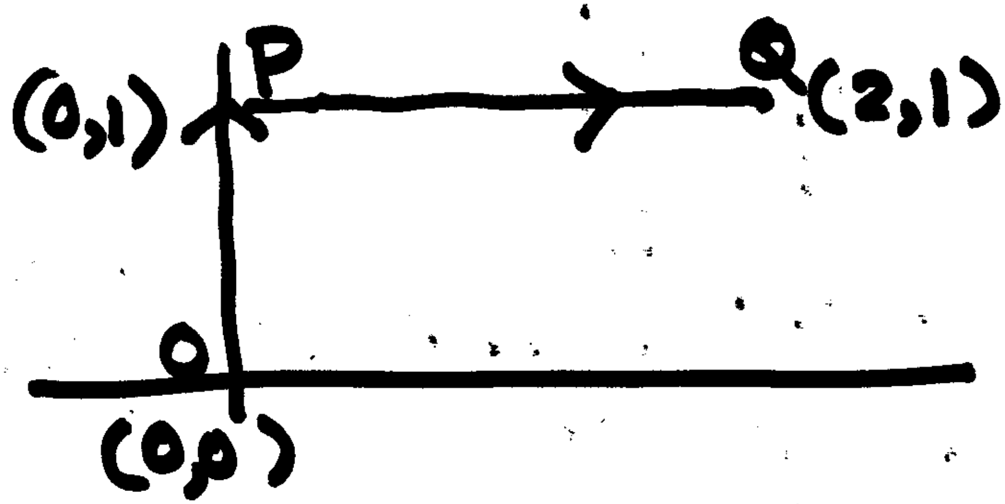
$$= \frac{3}{2} + \frac{4}{3} = \frac{17}{6}$$

$$y = \frac{x^2}{4} : x = 2\sqrt{y}$$

$$\int_0^2 \left(x^2 - \frac{x^4}{16} \right) dx + 2\int_0^1 2\,\underline{\underline{\sqrt{y}\cdot y}}\, dy$$

$$= \left(\frac{x^3}{3} - \frac{x^5}{80} \right) \Bigg|_0^2 + 4 \cdot \frac{y^{5/2}}{5/2} \Bigg|_0^1$$

$$= \frac{34}{15} + \frac{8}{5} = \frac{58}{15}$$



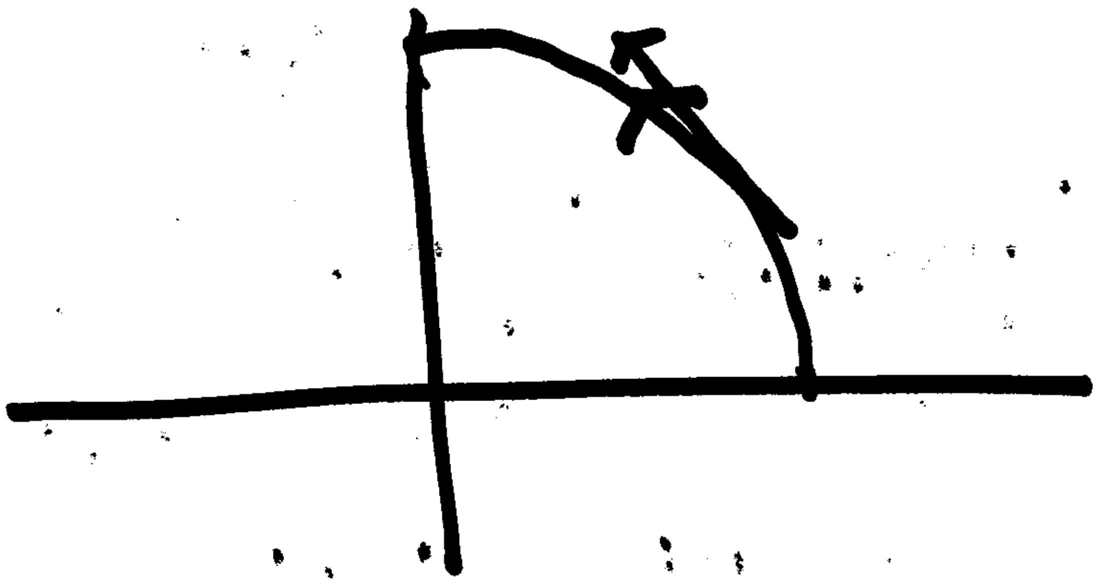
$$\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$$

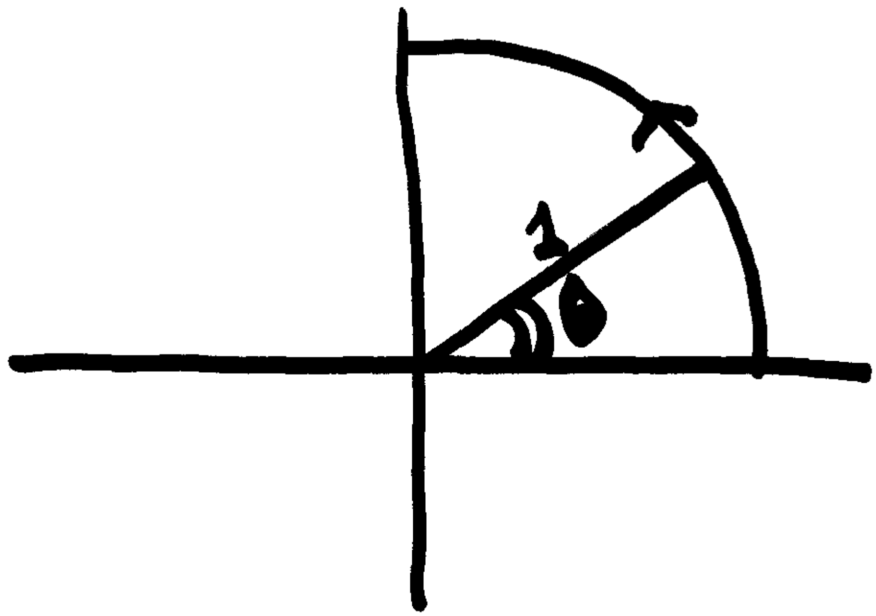
$O \rightarrow P$ ~~OP → PQ~~ : $x = 0 \quad dx = 0 \; ; \; I_1 = 0$

$P \rightarrow Q \quad y = 1 \; ; \; dy = 0$

$$\int_0^2 (x^2 - 1)\,dx = \left(\frac{x^3}{3} - x\right)_0^2 = \frac{8}{3} - 2 = \frac{2}{3}$$

$$\vec{F} = -y\,\hat{\imath} + x\,\hat{\jmath}$$





$x = \cos\theta$
$y = \sin\theta$

$dx = -\sin\theta \, d\theta$
$dy = \cos\theta . d\theta$

$\int (-y\,dx + x\,dy)$

$\int_C (+\sin^2\theta \, d\theta) + \int_C \cos^2\theta \, d\theta$

$= \int (\sin^2\theta + \cos^2\theta) d\theta = \int_0^{\pi/2} d\theta$

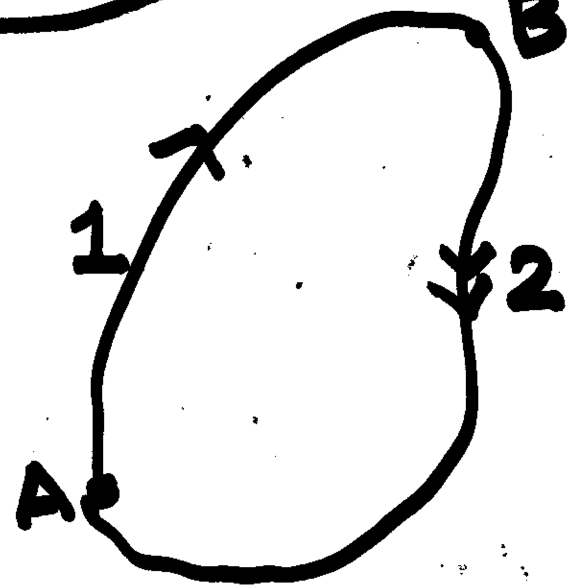
$= \pi/2$

$$\int_A^B \vec{F}\cdot d\vec{l}$$

$$\oint \vec{F}\cdot d\vec{l} = 0$$

$$\underset{1}{\int_A^B} \vec{F}\cdot d\vec{l} + \underset{2}{\int_B^A} \vec{F}\cdot d\vec{l}$$

$$\Downarrow$$



1 $$\int_A^B \vec{F} \cdot dl = \varphi(B) - \varphi(A)$$

2 $$\int_B^A \vec{F} \cdot dl = \varphi(A) - \varphi(B)$$

$$\oint \vec{F} \cdot \vec{dl} = 0$$

$$\vec{F} = \vec{\nabla}\phi$$

$$\int_A^B \vec{F}\cdot\vec{dl} = \int_A^B \left(\hat{\imath}\frac{\partial\phi}{\partial x} + \hat{\jmath}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}\right)\cdot \left(\hat{\imath}\,dx + \hat{\jmath}\,dy + \hat{k}\,dz\right)$$

$$= \int_A^B \left(\frac{\partial\phi}{\partial x}\cdot dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz\right)$$

$$= \int_A^B d\phi = \phi(B) - \phi(A)$$

$$\vec{F} = -\nabla\phi$$

Potential

$$\int \vec{F} \cdot \hat{n} \, ds$$

$$\int \underline{\underline{\vec{E}\cdot\hat{n}}}\, ds$$

Flux

$$\int \varphi \, dV = \iiint dx\, dy\, dz\, \varphi(x,y,z$$

$$\int \vec{F}\, dV = \int (\hat{\imath} F_x + \hat{\jmath} F_y + \hat{k} F_z) \, dV$$

$$= \hat{\imath} \underline{\underline{\int F_x \, dV}} + \hat{\jmath} \underline{\underline{\int F_y \, dV}} + \hat{k} \int F_z \, dV$$

$$\operatorname{div} \vec{F} = \vec{\nabla} \cdot \vec{F}$$

$$= \lim_{\Delta V \to 0} \frac{\oint \vec{F} \cdot \hat{n} \, ds}{\Delta V}$$