Lecture 26

Contents:
Exercise-1
Exercise-2
Exercise-3

Key Words: Fluid flow, Macroscopic Balance, Frictional Losses, Turbulent Flow, Venturimeter, Orifice Meter, Pitot Tube, Stack, Chimney, Draft, Natural draft

Exercise -1

Calculate velocity and flow rate of air leaking through an opening of rectangular cross section in a furnace wall (as shown below in the figure) from the following data:

![Diagram of air leakage through furnace opening](image)

**Figure 1: Air leakage through furnace opening**

Cross section of the opening \(0.10 \text{ m} \times 0.15 \text{ m}\)
Draft across the opening \(1.5 \text{ mm water}\)
Wall thickness \(0.52 \text{ m}\)

Assume turbulent flow and \(f = 0.0064\)
Friction losses due to contraction and expansion are 0.5 and 1 respectively.

Air is at $1.0133 \times 10^5$ N/m$^2$ pressure and 298 K temperature

**SOLUTION:**

Applying energy balance at plane 1 and 2

$P_1 = P_2$ (Atmospheric pressure)

$\Delta z = 0$ Since opening is horizontal

$M = 0$ No fan

$V_1 = V_2 = 0$ Velocity at both planes 1 and 2 $= 0$.

\[
\frac{d_2 - d_1}{\rho h} = -F \quad (1)
\]

\[
d_2 = 0 \therefore d_1 = \rho_h F \quad (2)
\]

Or \[
\frac{d_1}{\rho_h} = \bar{V}^2 \left[ 2f \cdot \frac{L}{D_e} + \frac{1}{2} \times ef_1 + \frac{1}{2} \times ef_2 \right] \quad (3)
\]

$\bar{V}$ is velocity of air in duct. $D_e$ is equivalent diameter

$D_e = \frac{2 \times 0.1 \times 0.15}{0.25} = 0.12$

$d_1 = 1.5 \times 9.860 \times 10^{-3} \text{ m}^2/\text{s}^2 \rho_{air} = 1.19 \text{ kg/m}^3$

Substituting the values in eq 3

$12.43 = 0.805 \bar{V}^2$

$\bar{V} = 3.93 \text{ m/s}$

Air flow rate $= 0.059 \text{ m}^3/\text{s}$

Now we can show that the flow is turbulent.

$Re = \frac{D_e \bar{V} \rho}{\mu} = 0.3 \times 10^5$; the flow is turbulent.

**Exercise-2**

A brick chimney 3.5m inside diameter (round) and 45m high is to handle flue gases (average molecular weight 30) at 603K. The atmospheric pressure outside the chimney is 734mm Hg and outside air is at 300K. It may be assumed that the gases do not cool as they rise in the chimney. Make the necessary calculations and prepare the following graphs:
a) Draft at the bottom of the chimney vs. flow rate of waste gases and

b) Horsepower equivalent of the flow energy available for draft at the bottom of the stack vs. flow rate of flue gases, Ignore the losses due to contraction and expanses of gases.

For both the plots an a and b, the graph should cover the entire range of chimney flow rates from 0 to the flow rate at which the available draft at the bottom of the chimney is nil

Use \( f = 0.0455 \left( \text{Re} \right)^{-0.2} \) and Viscosity of gas = \( 19.3 \times 10^{-7} \ T^{0.8} \ \text{g cm}^{-1} \ \text{s}^{-1} \) where \( T \) is in K.

**Solution**

a) Mechanical energy balance for flow of gases

\[
g (z_2 - z_1) - \frac{\rho_{\text{air}}}{\rho_f} (z_2 - z_1) g + \frac{\text{Draft}}{\rho_f} + F = 0.
\]

\( \rho_f = 0.586 \ \text{kg m}^{-3} \) and \( \rho_{\text{air}} = 1.178 \ \text{kg m}^{-3} \)

Substituting values and after simplification

\[
\text{Draft} = 261.33 - \rho_f F
\]

\[
F = 2f \left( \frac{L}{D} \right) V^2 = 2 \times 0.0455 \left( \frac{\rho_f V D}{\mu} \right)^{-0.2} \times \frac{L}{D} \times V^2
\]

Putting \( V = \frac{Q}{\pi D^2} \) and \( \mu = 3.23 \times 10^{-5} \ \text{kg m}^{-1} \ \text{s}^{-1} \) and other values of variables we get.

\[
\text{Draft} = 261.33 - 1.27 \times 10^{-3} Q^{1.8}
\]

We note at \( Q = 0 \), \( \text{draft} = 261.33 \).

And at \( Q = 895 \ \text{m}^3 \ \text{s}^{-1} \) \( \text{draft} = 0 \)

b) Flow energy in (W) = \[
261.33Q - 1.27 \times 10^{-3} Q^{2.8}
\]

This equation shows that flow energy will be maximum at \( Q = 505.95 \ \text{m}^3/\text{s} \) and zero at \( Q = 895.119 \ \text{m}^3/\text{s} \).

**Exercise-3**

A brick flue must be designed to discharge 425 m³/min (300K and 1 atm) of flue gas from furnace to stack. The flue is horizontal with a total length of 100 m and the four sharp 90 degree bends (L/D for one sharp bens is 20). The flue is rectangular in cross section with a 2:1 ratio of height to width. The average temperature of the flue gas is 350 degree C.

Calculate the following:
a) Pressure drop in mm water to be expected if the internal cross section of the flue were 120cm × 60 cm, 
b) Energy consumed by friction in the flue (watts) 
c) What would be the minimum cross-sectional dimensional of the flue if the pressure drop is limited to 2.5 mm of waste gases.

Use the following values:

Molecular weight of flue gas 29, \( e_{r1} = 0.4 \) and \( e_{r2} = 1.0 \)

Universal gas constant 8314kJ/kg mol × K and \( f = 0.0455 \) (Re)^{-0.2}

Viscosity 19.3x 10^{-7} T^{0.8} g cm^{-1} sec^{-1}, where T is in K, 1 N/m^2 = 0.102 mm of water

SOLUTION:

a) Mechanical energy balance gives

\[
(P_1 - P_2) = \frac{\rho V^2}{2} \left( e_{f1} + 4f \frac{L}{D} + e_{f2} \right)
\]

\[
\frac{L}{D} = \left( \frac{L}{D_e} \right)_{pipe} + 4 \left( \frac{L}{D_e} \right) \quad D_e = 80 \text{ cm.}
\]

\[
= \frac{100}{0.8} + 80 = 205
\]

\[
\bar{V} = 20.4 \text{ m/s} \quad \text{and} \quad f = 0.00371
\]

\[
\rho = 0.563 \text{ kg m}^{-3}
\]

Substituting the values we get

\[
(P_1 - P_2) = 521.42 \text{ N/m}^2 = 53.18 \text{ mm H}_2\text{O}
\]

b) Energy consumed by friction = \( W \times F \).

Where \( F = \frac{V^2}{2} \left( e_{f1} + 4f \left( \frac{L}{D} \right) + e_{f2} \right) \).

Energy = \[
\frac{623}{300} \times \frac{425}{60} \times 0.563 \times \frac{20.42^2 \times 20.42}{2} \times 4.4422
\]

= 7670W.

c) Let the height of the rectangular cross section is \( h \)

Width = \( h/2 \).

\[
D_e = \frac{2h}{3}
\]

Mechanical energy balance
\[ \frac{P_1 - P_2}{\rho} = 2 f \frac{L}{D} \bar{V}^2 + \frac{3.32 \times 10^5}{h} \]

Substituting the values

\[ \frac{24.5}{0.563} = \frac{864.36}{h^4} \left[ 2 \times 0.0455 \left( \frac{3.32 \times 10^5}{h} \right)^{-0.2} \times \frac{3 \times 205}{2h} + 0.7 \right] \]

Solving, we get,

\[ h \approx 2.46 \text{ m} \]

width = 1.23 m

Cross section of flue = 2.46 \times 1.23 m

**ASSIGNMENT:**

1) Calculate velocity and flow rate of air leaking through an opening of rectangular cross section in a furnace wall (as shown below in the figure) from the following data:

**Figure 1: Air leakage through furnace opening**

Cross section of the opening = 0.10 m \times 0.15, Draft across the opening = 1.5 mm water

Wall thickness = 0.52 m, Assume turbulent flow and \( f = 0.0064 \)

Friction losses due to contraction and expansion are 0.5 and 1 respectively.

Air is at 1.0133 \times 10^5 \text{ N/m}^2 \text{ pressure and 298 K temperature}

2) A brick flue must be designed to discharge 425 m$^3$/min (300K and 1 atm) of flue gas from furnace to stack. The flue is horizontal with a total length of 100 m and the four sharp 90 degree bends (L/D for one sharp bend is 20). The flue is rectangular in cross section with a 2:1 ratio of height to width. The average temperature of the flue gas is 350 degree C.

Calculate the following:

- d) Pressure drop in mm water to be expected if the internal cross section of the flue were 120cm \times 60 cm,
- e) Energy consumed by friction in the flue (watts)
- f) What would be the minimum cross-sectional dimensional of the flue if the pressure drop is limited to 2.5 mm of waste gases.

Use the following values: Molecular weight of flue gas 29, \( e_{f1} = 0.4 \) and \( e_{f2} = 1.0 \), Universal gas constant \( \frac{8314 \text{kJ}}{\text{mol} \times \text{K}} \) and \( f = 0.0455 \left( \text{Re} \right)^{-0.2}, \text{Viscosity} 19.3 \times 10^{-7} T^{0.8} \text{ g cm}^{-1} \text{ sec}^{-1} \), where \( T \) is in kelvin, 1 N/m$^2$ = 0.102 mm of water.