Department Of
MECHANICAL ENGINEERING

MODULE 10

Mass Transfer

What is Mass Transfer?

"Mass transfer specifically refers to the relative motion of species in a mixture due to concentration gradients."

Analogy between Heat and Mass Transfer

Since the principles of mass transfer are very similar to those of heat transfer, the analogy between heat and mass transfer will be used throughout this module.

Mass transfer through Diffusion

Conduction

$$q'' = -k\frac{dT}{dy}\left[\frac{J}{m^2 s}\right]$$

(Fourier's law)

Mass Diffusion

$$j''_A = -\rho D_{AB}\frac{d\xi_A}{dy}\left[\frac{kg}{m^2 s}\right]$$

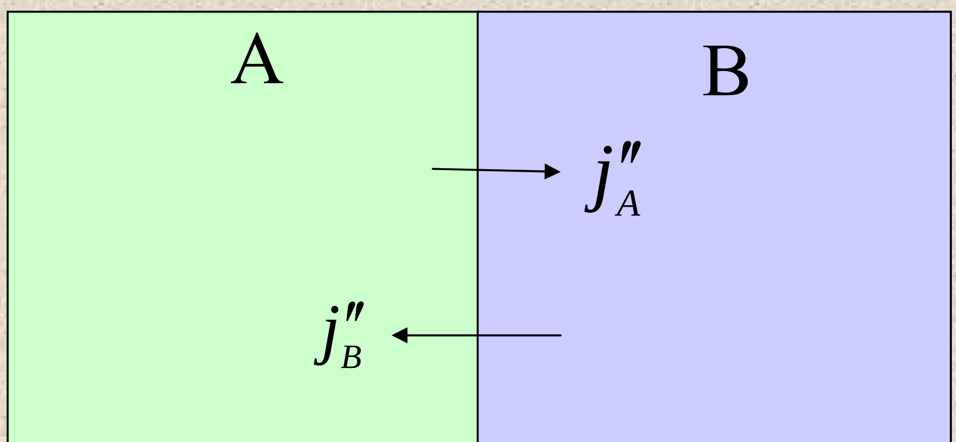
(Fick's law)

ρ is the density of the gas mixture

DAB is the diffusion coefficient

$\xi_A = \rho_A / \rho$ is the mass concentration of component A

Mass transfer through Diffusion



The sum of all diffusion fluxes must be zero: $\sum j''_i = 0$

$$\xi_A + \xi_B = 1$$

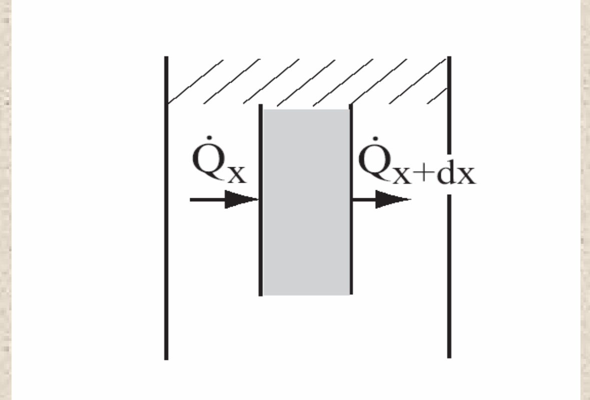
$$\frac{d}{dy}\xi_A = -\frac{d}{dy}\xi_B$$

$$D_{BA} = D_{AB} = D$$

Heat and Mass Diffusion: Analogy

- Consider unsteady diffusive transfer through a layer

Heat conduction, unsteady, semi-infinite plate



$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c}\frac{\partial^2 T}{\partial x^2} = \alpha \frac{\partial^2 T}{\partial x^2}$$

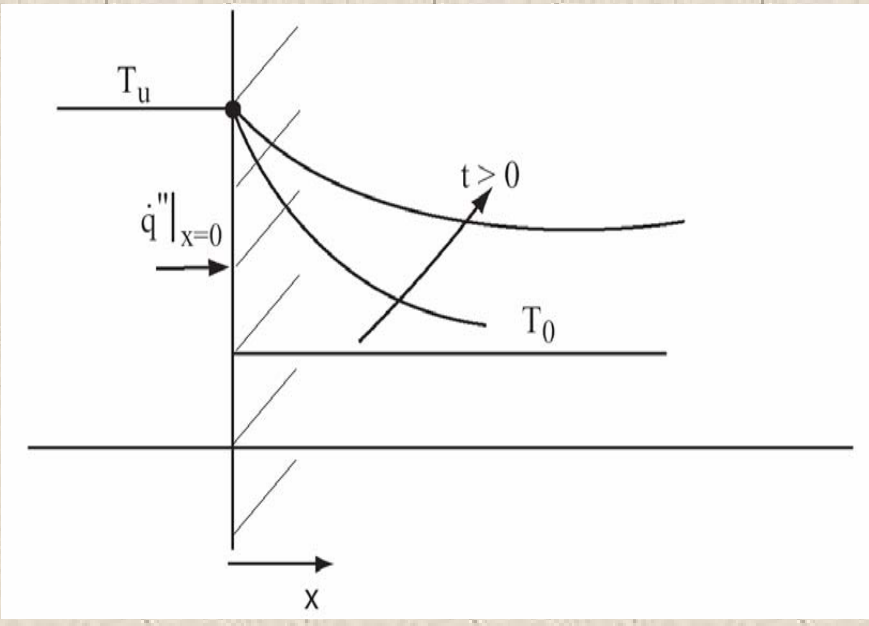
Similarity transformation:
PDE → ODE

$$\frac{d^2\theta}{dy^2} + 2\eta \frac{d\theta^*}{d\eta} = 0 \qquad \eta = \frac{x}{\sqrt{4\alpha t}}$$

Heat and Mass Diffusion: Analogy

Solution: $$\frac{T - T_0}{T_u - T} = 1 - erf\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

Temperature field



Heat flux $$\dot{q}''\Big|_{x=0} = -k\frac{dT}{dx}\Big|_{x=0} = \frac{k}{\sqrt{\pi\alpha t}}\left(T_u - T_0\right) = \sqrt{\frac{kc\rho}{\pi t}}\left(T_u - T\right)$$

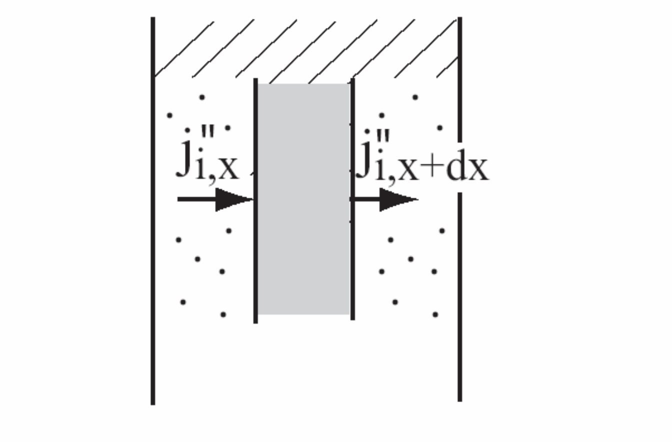
Heat and Mass Diffusion: Analogy

Diffusion of a gas component, which is brought in contact with another gas layer at time t=0

Differential equation:

$$\frac{\partial \rho_i}{\partial t} = \rho D \frac{\partial^2 \xi_i}{\partial x^2}$$

$$\frac{\partial \xi_i}{\partial t} = D \frac{\partial^2 \xi_i}{\partial x^2}$$



Transient diffusion

Initial and boundary conditions:

$$\xi_i(t=0, x) = \xi_{i,o}$$

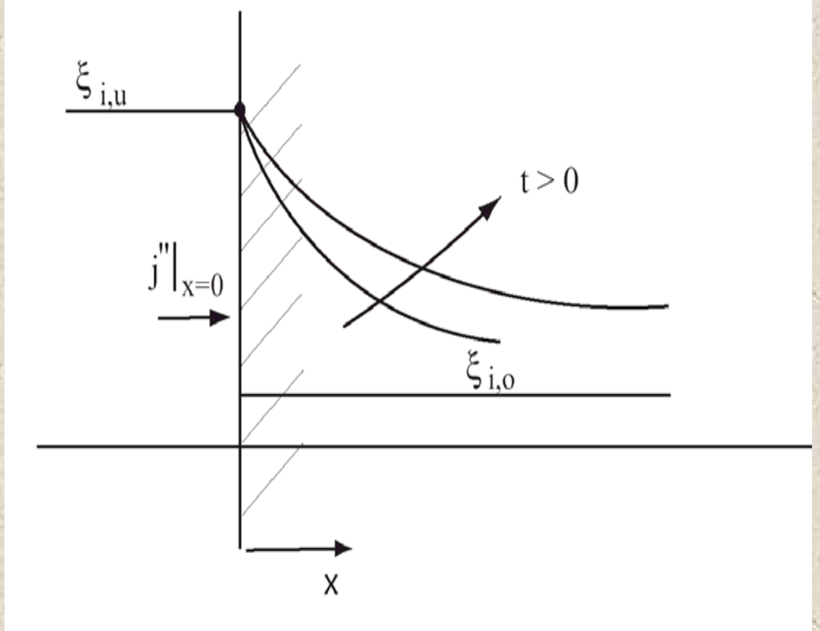
$$\xi_i(t>0, x=0) = \xi_{i,u}$$

$$\xi_i(t>0, x\to\infty) = \xi_{i,o}$$

Heat and Mass Diffusion: Analogy

Solution: $$\frac{\xi_i - \xi_{i,o}}{\xi_u - \xi_{i,o}} = 1 - erf\left(\frac{x}{\sqrt{4Dt}}\right)$$

Concentration field



Diffusive mass flux $$j_i''\Big|_{x=0} = \frac{\rho D}{\sqrt{\pi D t}}\left(\xi_{i,Ph} - \xi_{i,o}\right)$$

Diffusive mass transfer on a surface (Mass convection)

Fick's Law, diffusive mass flow rate:

$$j''_A = -\rho D \frac{\partial \xi}{\partial y}\Big|_{y=0} = -\rho D \frac{\xi_\infty - \xi_w}{L} \frac{\partial \xi^*}{\partial y^*}\Big|_{y^*=0}$$

mass transfer coefficient $h_{mass}\left[\frac{kg}{m^2 s}\right]$

$$j''_A = h_{mass}\left(\xi_w - \xi_\infty\right)$$

Dimensionless mass transfer number, the **Sherwood number** Sh $\frac{h_{mass} L}{\rho D} = \mathrm{Sh} = \frac{\partial \xi^*}{\partial y^*}\Big|_{y^*=0} = f(\mathrm{Re}, \mathrm{Sc})$

$$Sh = C \,\mathrm{Re}^m \, Sc^n$$

Note: Compare with energy eqn. and Nusselt No.: The constants *C* and the exponents *m* and *n* of both relationships must be equal for comparable boundary conditions.

Diffusive mass transfer on a surface..

Dimensionless number to represent the relative magnitudes of heat and mass diffusion in the thermal and concentration boundary layers

Lewis No.

$$\mathrm{Le} = \frac{\mathrm{Sc}}{\mathrm{Pr}} = \frac{\alpha}{D} = \frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}}$$

Analogy between heat and mass transfer

Comparing the correlation for the heat and mass transfer

$$\frac{Sh}{Nu} = \left(\frac{Sc}{\mathrm{Pr}}\right)^{n}$$

Hence,

$$\frac{h_{mass}}{h / c_p} = \left(\frac{Sc}{\mathrm{Pr}}\right)^{n-1}$$

For gases, Pr ≈ Sc, hence: $\frac{h_{mass}}{h / c_p} = 1$ **Lewis relation**