Coupled Electrothermal-elastic Modelling

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Effects of heating on mechanical deformation

\[ \alpha = \frac{d\varepsilon}{dT} \]  
Temperature coefficient of expansion

\[ \varepsilon(T) = \varepsilon(T_0) + \alpha(T - T_0) \]  
Uniaxial thermal strain

\[ \varepsilon_{\text{mismatch}}(T) = (\alpha_f - \alpha_s)(T - T_0) \]  
Mismatched thermal strain and stress between a film and a substrate that are bonded to each other.

\[ \sigma_{\text{mismatch}} = \left( \frac{E}{1 - v} \right) \varepsilon_{\text{mismatch}} \]

\[ \varepsilon_z = -\left\{ \alpha_f + 2v(\alpha_f - \alpha_s) \right\}(T - T_0) \]  
Total strain for a sandwiched film in the thickness (z) direction
Embedded actuation:
Actuator and mechanism are together.
(Guckel et al., 1992; Comtois and Bright, 1996)

Series connection

Bends up

TEMP VALUE
+3.0 E+02
+3.19 E+02
+3.57 E+02
+3.56 E+02
+3.74 E+02
+3.93 E+02
+4.11 E+02
+4.30 E+02
+4.48 E+02
+4.67 E+02

cold

hot

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Parallel connection

\[ R = \frac{\ell}{A} \]

Temperature distribution.
Selective doping (if made with silicon)
ETC expansion block
Parallel micro manipulator

With three degrees of freedom; Made using MUMPs, polysilicon.
Devices made with PennSOIL

Doped

SiO₂

Silicon

Silicon

1 mm
Modeling
Governing equations

**Electrical Domain**

\[ \nabla \cdot (\tilde{k}_e \nabla v) + i_e = 0 \quad \text{in } \Omega \]

\[ \nu = \nu_e \quad \text{on } \Gamma_{eE} \]

\[ \bar{n} \cdot (\tilde{k}_e \nabla v) = f_e \quad \text{on } \Gamma_{nE} \]

**Elastic Domain**

\[ \nabla \bar{\sigma} + \bar{F} = 0 \quad \text{in } \Omega \]

\[ \bar{\sigma} = \tilde{E} [\tilde{\varepsilon} - \alpha(T - T_0) \bar{I}] \quad \text{in } \Omega \]

\[ \tilde{\varepsilon} = \frac{\nabla \bar{u} + (\nabla \bar{u})^T}{2} \quad \text{in } \Omega \]

\[ \bar{u} = \bar{u}_e \quad \text{on } \Gamma_{eM} \]

\[ \bar{n} \cdot \bar{f}_u = \bar{f}_u \quad \text{on } \Gamma_{nM} \]

**Thermal Domain**

\[ \nabla \cdot (\tilde{k}_t \nabla T) + \dot{q}_T = 0 \quad \text{in } \Omega \]

\[ \dot{q}_T = \tilde{k}_e \nabla \nu \cdot \nabla \nu \quad \text{in } \Omega \]

\[ T = T_e \quad \text{on } \Gamma_{eT} \]

\[ \bar{n} \cdot (\tilde{k}_t \nabla T) = f_T \quad \text{on } \Gamma_{nT} \]

**Inter-domain Coupling**

\[ \tilde{k}_e(T), \quad q_T(v), \quad \bar{E}(T), \quad \alpha(T), \]

**Nonlinearity**

\[ \tilde{k}_e(T), \quad \tilde{k}_t(T), \quad f_T(T), \]

\[ \bar{E}(T), \alpha(T). \]
Thermal modeling

- **Convection**
  - Temperature dependence of heat transfer properties.
  - Size dependence of heat transfer properties.

- **Radiation**
  - View / Shape factors.
  - Radiation heat transfer between parts of the same device at different temperatures.

- **Boundary Conditions**
  - Essential Boundary conditions at the device anchor.
  - Natural Boundary conditions at the device anchor.

- **Conduction through trapped air volume**
  - Conduction between parts of the same device at different temperature with an intervening trapped air volume.
  - Conduction from the underside of the device to the substrate through the air trapped between them.

- **Temperature dependence of thermo-physical Properties**

\[ h = \text{heat transfer coefficient} \]
Why convection and radiation?

Thermal Expansion Device (TED), Cragun & Howell (1998)

Without convection or radiation

With convection and radiation
EBC v/s NBC

**Essential** Boundary C
*Thermally Grounded*

**Natural** Boundary C
*Not Thermally Grounded*
The Finite Element model

20 node, 3-D Continuum elements in ABAQUS

Fully Coupled Electro-Thermal Analysis

Sequentially Coupled Thermo-Elastic Analysis

With temperature dependent material properties and heat transfer coefficients.
Thermal Boundary Conditions and Scaling: Case Studies

- **Same Maximum Temperature at Steady State**
  - EBC + Meso
  - NBC + Meso
  - EBC + Micro
  - NBC + Micro

  Made using PennSOIL

- **Same Power Input**
  - EBC + Meso
  - NBC + Meso
  - EBC + Micro
  - NBC + Micro

  Made using MUMPs

- **Same Applied Voltage**
  - EBC + Meso
  - NBC + Meso
  - EBC + Micro
  - NBC + Micro

Experiment

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Same Maximum Temperature

Meso / EBC

Meso / NBC

Micro / EBC

Micro / NBC

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Same Maximum Temperature

Meso 10 times

Micro

Displacement
Same Applied Voltage

Maximum Temperature

Normalised Transverse Displ.

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Meso scale EBC

Meso scale NBC

Tip deflection (μm)

Applied Voltage V

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More complicated geometry

Temperature K

Applied Voltage V

deflection um

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One dimensional approximation

Electrical Model

Thermal Model

Elastic Model

(Maizel’s theorem to compute the output deflection)
Parameters for analytical modeling of electro-thermal-compliant actuator

Out-of-plane thickness $= p_3 L$

$L = \delta z_e$
Electrical analysis

Notice how resistance, current, and dissipated power vary with scaling.
Coupling between electrical and thermal analysis

\[ \dot{Q}_{e_i} = \frac{J^2 R_i}{A_i L_i} = \frac{L^2 V^2}{\phi_e^2 A_i^2 \rho_e} \quad i = 1, 2, 3, 4 \]

\[ \dot{Q}_{e_1} = \frac{V^2}{\phi_e^2 p_1^2 p_2^2 L^2 \rho_e} \]

\[ \dot{Q}_{e_2} = \frac{V^2}{\phi_e^2 p_1^2 p_2^2 L^2 \rho_e} \]

\[ \dot{Q}_{e_3} = \frac{V^2}{\phi_e^2 p_1^2 p_3^2 L^2 \rho_e} \]

\[ \dot{Q}_{e_4} = \frac{V^2}{\phi_e^2 p_1^2 p_2^2 L^2 \rho_e} \]
Thermal analysis

Temperature profile in the connector is not modeled as it is negligibly short.

Out-of-plane thickness = $pL$

\[
\frac{d^2 T_i(x)}{dx^2} + \frac{\dot{Q}_e}{k_t} = 0 \quad i = 1, 3, 4
\]

\[
T_i(x) = -\frac{\dot{Q}_e}{2k_t} x^2 + a_i x + b_i \quad i = 1, 3, 4
\]

Six constants to be evaluated from the boundary conditions.
Boundary conditions to solve for constants

\[ T_i(x) = -\frac{\dot{Q}_{e_i}}{2k_t}x^2 + a_ix + b_i \quad i = 1, 3, 4 \]

1. Temperature raise at the left end is zero.
\[ T_1(x = 0) = T_0 \quad \Rightarrow \quad b_1 = T_0 \]

2. Temperature raises at the interface of first and third segments are equal.
\[ T_1(x = L_1) = T_3(x = 0) \quad \Rightarrow \quad -\frac{\dot{Q}_{e_1}}{2k_t}L^2 + a_1L + T_0 = b_3 \]

3. Thermal equilibrium of the second segment
\[-k_tA_1 \left. \frac{dT_1}{dx} \right|_{x=L_1} - k_tA_3 \left. \frac{dT_3}{dx} \right|_{x=0} + \dot{Q}_{e_2}A_2L_2 = 0\]
\[-k_t p_t p_2 L^2 \left( -\frac{\dot{Q}_{e_1}}{k_t}L + a_1 \right) - k_t p_t p_3 L^2 a_3 + \dot{Q}_{e_2} p_t p_2^3 L^3 = 0\]
Boundary conditions (contd.)

\[ T_i(x) = -\frac{\dot{Q}_{e_i}}{2k_t} x^2 + a_i x + b_i \quad i = 1, 3, 4 \]

4. Temperature raises at the interface of third and fourth segments are equal.

\[ T_3(x = L_3) = T_4(x = 0) \quad \Rightarrow \quad \frac{\dot{Q}_{e_3}}{2k_t} (1 - p_1)^2 L^2 + a_3 (1 - p_1) L + b_3 = b_4 \]

5. Heat flux continuity at the interface of third and fourth segments.

\[ k_t A_3 \frac{dT_3}{dx} \bigg|_{x=L_3} - k_t A_4 \frac{dT_4}{dx} \bigg|_{x=0} = 0 \quad \Rightarrow \quad k_t p_t p_3 L^2 \left( -\frac{\dot{Q}_{e_3}}{k_t} (1 - p_1) L + a_3 \right) - k_t p_t p_2 L^2 a_4 = 0 \]

6. Temperature raise at the end of the fourth segment is zero.

\[ T_4(x = L_4) = T_0 \quad \Rightarrow \quad p_1 L a_4 + b_4 = \frac{\phi_{i_5} V^2}{\phi_e \rho_e k_t} + T_0 = c_5 \]

Total temperature raise = \[ \frac{V^2}{\rho_e k_t} \]

Notice the lack of scaling effect!
Elastic analysis

Maizel’s theorem to find the vertical deflection at the tip.

\[ \Delta = \sum_{i=1}^{4} \left[ \int_{0}^{L_i} \hat{F}_{axial_i}(x) \alpha \left\{ T(x)_i - T_0 \right\} dx \right] \]

\[ \frac{\Delta}{L} = \phi_\Delta V^2 \frac{\alpha}{\rho e k_i} \]

Notice on what quantities the relative deflection depends.

With convection included, it would be different.
With convection included…

Without convection…

\[
\frac{\Delta}{L} = \phi \frac{V^2}{\rho_e k_t} \propto L^0
\]

With convection…

\[
\frac{\Delta}{L} \propto \frac{1}{\sqrt{L}}
\]

\[
T \propto \frac{\dot{Q}L}{h} \propto \frac{V^2}{\rho_e hL} \propto \frac{1}{L}
\]

\[
\frac{d^2 T_i(x)}{dx^2} + \frac{\dot{Q}_{e_i}}{k_t} = 0 \quad i = 1..4
\]
Main points

- Electro-thermal-elastic actuation is easy to implement in practice.
  - Large forces and displacements (relatively)
  - Slow up to $10^4$ s

- Coupled modelling is sequential, usually...
  - Temperature-dependent properties make it nonlinear.

- Reduced order modelling is convenience and useful.
  - But not applicable always.

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