COMPUTATIONAL HEAT TRANSFER AND FLUID FLOW

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Introduction

Outline

- Basics of Heat Transfer
- Mathematical description of fluid flow and heat transfer: conservation equations for mass, momentum, energy and chemical species, classification of partial differential equations generalized control volume approach in Eulerian frame

HEAT TRANSFER BASICS

What is Heat Transfer?

"Energy in transit due to temperature difference."

Thermodynamics tells us:

- How much heat is transferred (δQ)
- How much work is done (δW)
- Final state of the system

Heat transfer tells us:

- How (by which mechanism) δQ is transferred
- At what rate δQ is transferred
- Temperature distribution within the body

MODES

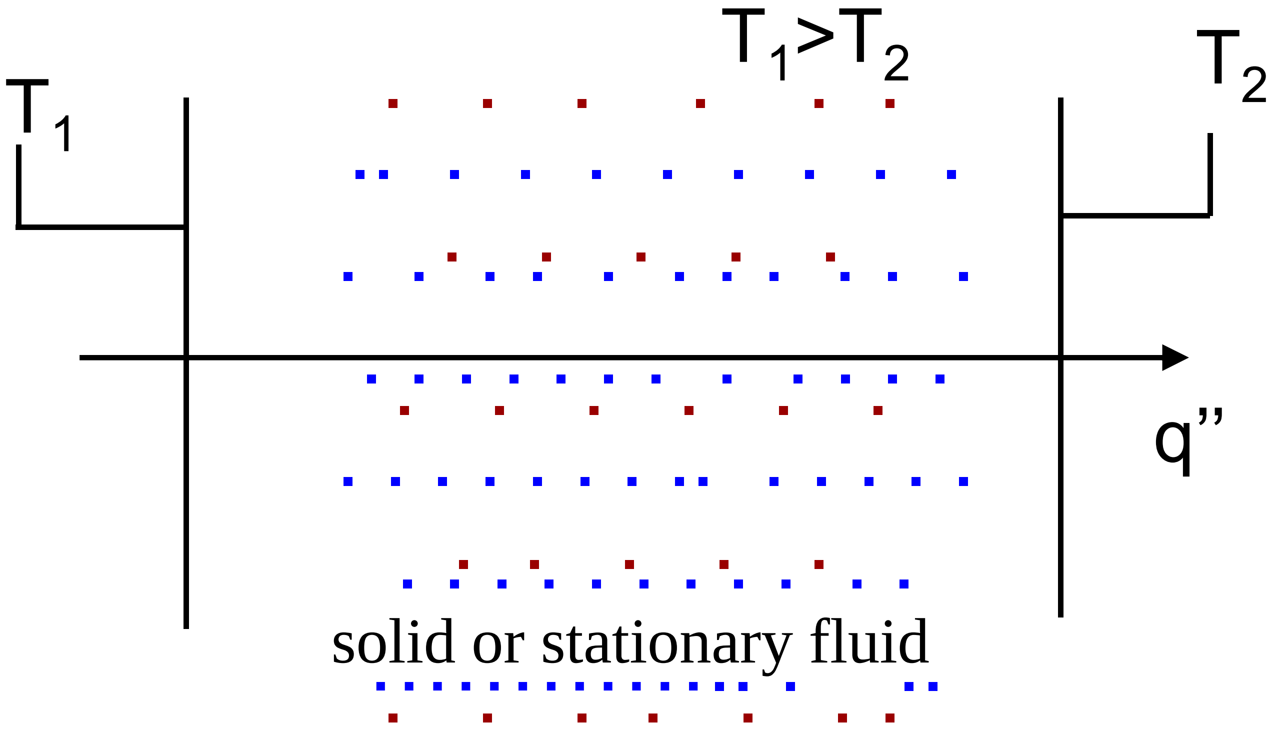
- ✓ Conduction
  - needs matter
  - molecular phenomenon (diffusion process)
  - without bulk motion of matter
- ✓ Convection
  - heat carried away by bulk motion of fluid
  - needs fluid matter
- ✓ Radiation
  - does not need matter
  - transmission of energy by electromagnetic waves

APPLICATIONS OF HEAT TRANSFER

- ✓ Energy production and conversion
  - steam power plant, solar energy conversion etc.
- ✓ Refrigeration and air-conditioning
- ✓ Domestic applications
  - ovens, stoves, toaster
- ✓ Cooling of electronic equipment
- ✓ Manufacturing / materials processing
  - welding, casting, forming, heat treatment, laser machining
- ✓ Automobiles / aircraft design
- ✓ Nature (weather, climate etc)

CONDUCTION

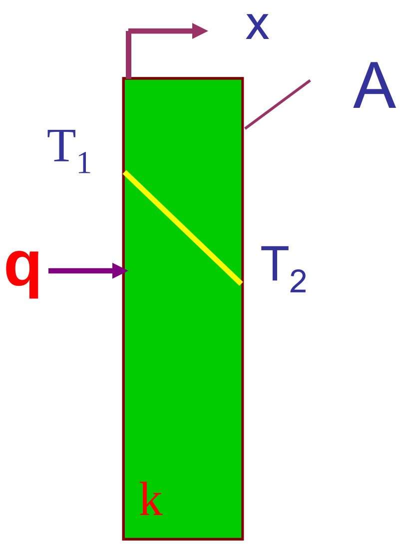
(Needs medium, Temperature gradient)



RATE:

q(W) or (J/s)   (heat flow per unit time)

Conduction (contd…)



Rate equations (1D conduction):

- Differential Form

  $q = - k A \, dT/dx$, W

  k = Thermal Conductivity, W/mK

  A = Cross-sectional Area, m^2

  T = Temperature, K or ^oC

  x = Heat flow path, m

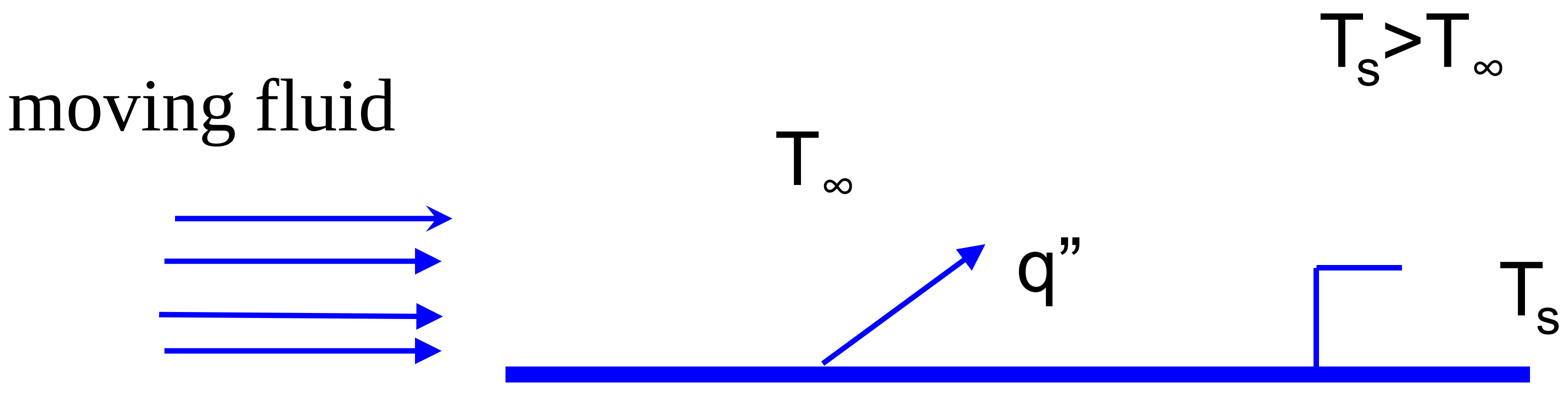
- Difference Form

  $q = - k A (T_2 – T_1) / (x_2 – x_1)$

Heat flux: $q'' = q / A = - k dT/dx$ (W/m^2)

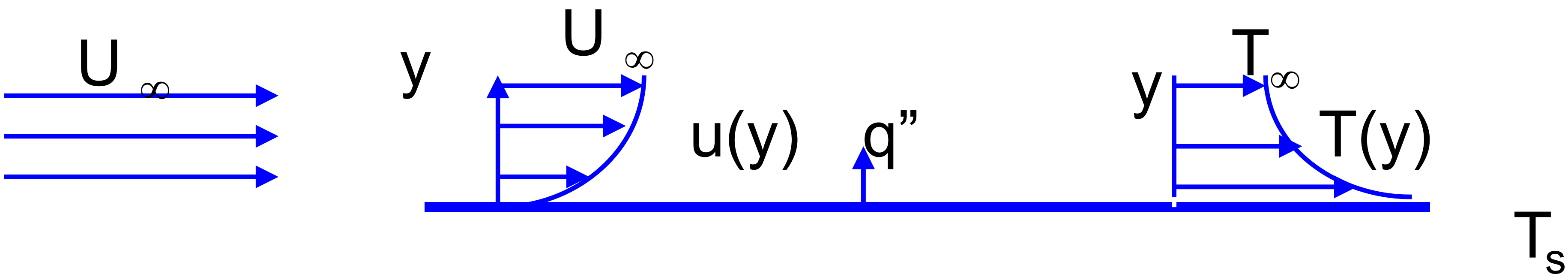
(negative sign denotes heat transfer in the direction of decreasing temperature)

CONVECTION



❖ Energy transferred by diffusion + bulk motion of fluid

Rate equation (convection)



Heat transfer rate $q = hA(T_s - T_\infty)$ W

Heat flux $q'' = h(T_s - T_\infty)$ W / m^2

h=heat transfer co-efficient (W /m^2K)

not a fluid property alone, but also depends on flow geometry, nature of flow (laminar/turbulent), thermodynamics properties etc.

Convection (contd…)

convection

- Free or natural convection (induced by buoyancy forces)
- Forced convection (induced by external means)

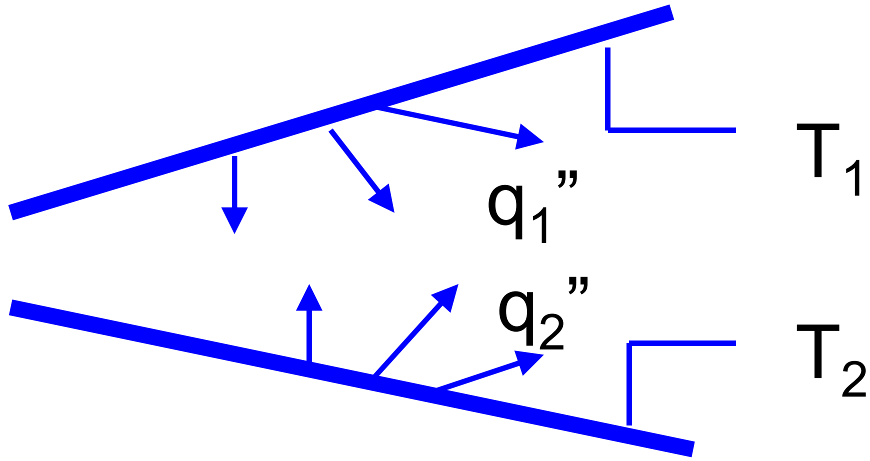
May occur with phase change (boiling, condensation)

Convection (contd…)

Typical values of h (W/m^2K)

Free convection	gases: 2 - 25
	liquid: 50 - 100
Forced convection	gases: 25 - 250
	liquid: 50 - 20,000
Boiling/Condensation	2500 -100,000

RADIATION



RATE:

q(W) or (J/s) Heat flow per unit time.

Flux : q'' (W/m^2)

Rate equations (Radiation)

RADIATION:

Heat Transfer by electro-magnetic waves or photons(no medium required.)

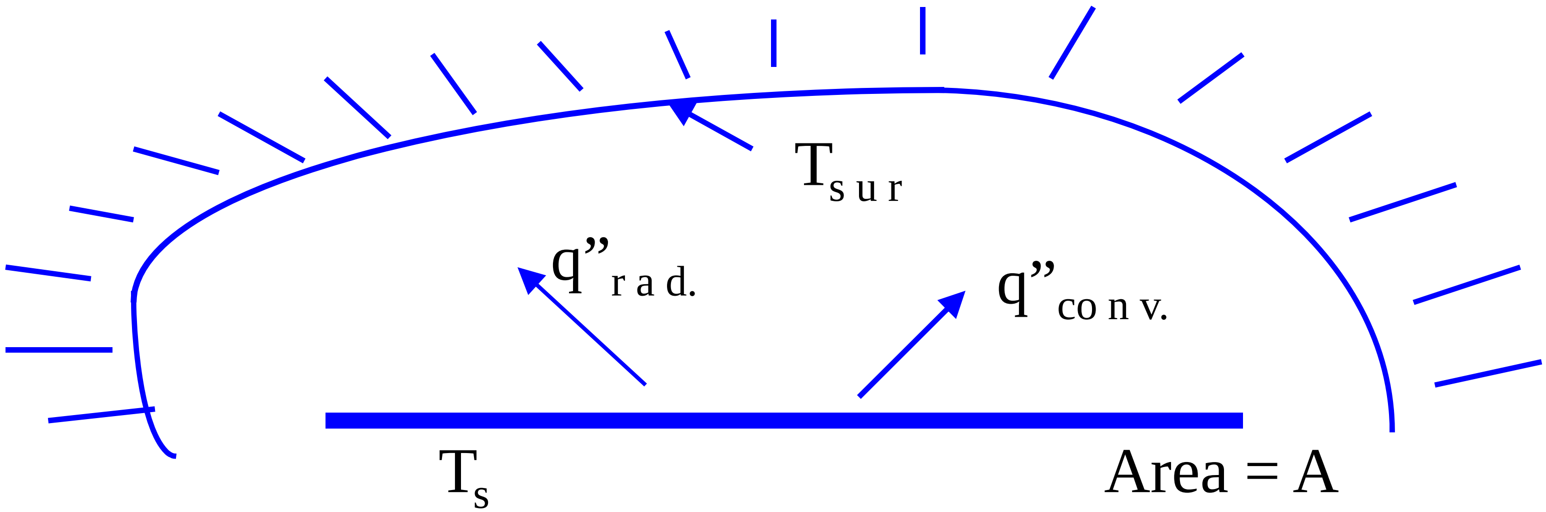
Emissive power of a surface (energy released per unit area):

$$E=\varepsilon\sigma T_s^4 \ (W/\ m^2)$$

ε= emissivity (property)……..

σ=Stefan-Boltzmann constant

Rate equations (Contd….)



Radiation exchange between a large surface and surrounding

$$Q''_{rad} = \varepsilon\sigma(T_s^4 - T_{sur}^4) \text{ W/ m}^2$$

Substantial derivative

$$\phi = \phi(x_1, x_2, x_3, t)$$

$$\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial \phi}{\partial x_2}\frac{dx_2}{dt} + \frac{\partial \phi}{\partial x_3}\frac{dx_3}{dt}$$

$$\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x_i}\frac{dx_i}{dt}$$

$$\underbrace{\frac{D\phi}{Dt}}_{\substack{\text{substantial}\\ \text{derivative}}} = \underbrace{\frac{\partial \phi}{\partial t}}_{\substack{\text{local}\\ \text{component}}} + \underbrace{u_i \frac{\partial \phi}{\partial x_i}}_{\substack{\text{convective}\\ \text{component}}}$$

Conservation principle(s)

- mass is conserved
- momentum is conserved
- energy is conserved
- chemical species are conserved
- …

$$\varphi = \varphi\left(x_1, x_2, x_3, t\right)$$

$$\Phi = \int_V \varphi dV$$

$$\underbrace{\frac{D\Phi}{Dt}}_{change} = \frac{D}{Dt}\int_V \varphi dV = \int_V \frac{D}{Dt}\left(\varphi dV\right) = \underbrace{\int_V \psi dV}_{source}$$

Conservation principle

for an elementary volume dV :

$$\frac{D}{Dt}(\varphi\, dV) = \psi\, dV$$

$$\frac{D}{Dt}(\varphi dV) = dV\,\frac{D\varphi}{Dt} + \varphi\,\frac{D}{Dt}(dV)$$

$$= dV\left(\frac{D\varphi}{Dt} + \varphi\,\frac{\frac{D}{Dt}(dV)}{dV}\right)$$

$$= \left(\frac{D\varphi}{Dt} + \varphi\frac{\partial u_i}{\partial x_i}\right)dV$$

Conservation principle

$$\frac{D}{Dt}\left(\varphi dV\right)=\left(\frac{D\varphi}{Dt}+\varphi\frac{\partial u_i}{\partial x_i}\right)dV$$

$$=\left(\frac{\partial \varphi}{\partial t}+u_i\frac{\partial \varphi}{\partial x_i}+\varphi\frac{\partial u_i}{\partial x_i}\right)dV$$

$$=\left[\frac{\partial \varphi}{\partial t}+\frac{\partial}{\partial x_i}\left(\varphi u_i\right)\right]dV$$

Conservation principle

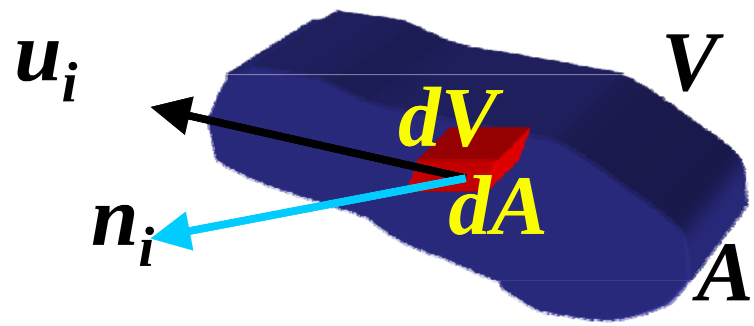
for an elementary volume dV *:*

$$\frac{D\varphi}{Dt} + \varphi \frac{\partial u_i}{\partial x_i} = \psi$$

$$\frac{\partial \varphi}{\partial t} = -\frac{\partial}{\partial x_i}(\varphi u_i) + \psi$$

integrated for a final volume V

bounded by a surface A *:*



$$\int_V \frac{D}{Dt}(\varphi dV) = \int_V \frac{\partial \varphi}{\partial t} dV + \int_V \frac{\partial}{\partial x_i}(\varphi u_i) dV =$$

$$= \int_V \frac{\partial \varphi}{\partial t} dV + \int_A \varphi n_i u_i dA = \int_V \psi dV$$

called as "Reynolds Transport Equation (RTE)" - a relation between a system and control volume.

Conservation of mass

Total mass remains constant. For a control volume, the balance of mass flux gives the mass accumulation rate.

$$\frac{D}{Dt}\left(\rho dV\right) = 0$$

set :

$\varphi = \rho$ - density, then

$\phi = \int_V \rho dV$ - mass

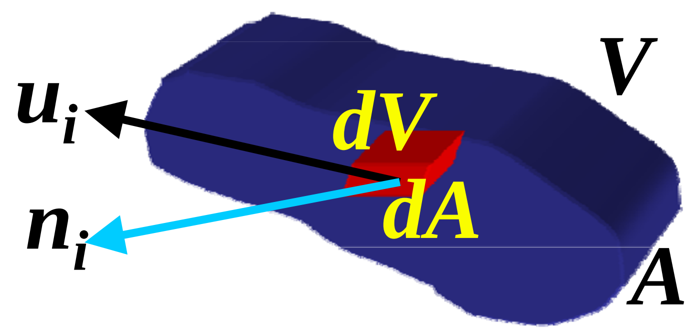
$\psi = 0$ - no source

Conservation of mass

continuity equation

for an elementary volume dV:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x_i}(\rho u_i)$$



integrated for a final volume V bounded by a surface A :

$$\int_V \frac{\partial \rho}{\partial t} dV = -\int_A \rho n_i u_i dA$$

Here (n_i u_i) is a scalar quantity:
- -for outward flow (away from the surface)- positive
- -for inward flow- negative

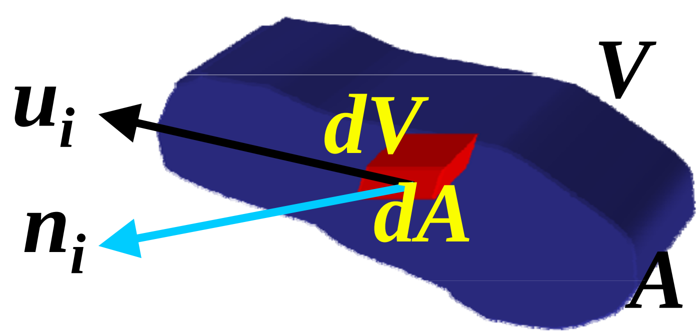
Incompressible flow

for an incompressible fluid: $\rho = const$ *:*

written for an elementary volume dV *:*

$$\frac{\partial u_i}{\partial x_i} = 0$$

or a final volume V *bounded by a surface* A *:*



$$\int_A n_i u_i dA = 0$$

Conservation of momentum

Rate of change of momentum will cause fluid acceleration.
For a control volume, we consider the balance of momentum fluxes.

$$\frac{D}{Dt}\left(\rho dV u_j\right) = F_j$$

$$\underbrace{\rho dV}_{mass}\ \underbrace{\frac{Du_j}{Dt}}_{acceleration} = \underbrace{F_j}_{force}$$

set :

$$\varphi = \rho u_j \quad \text{- density} \times \text{velocity, then}$$

$$\phi = \int_V \rho u_j dV \quad \text{- momentum}$$

$$\psi = F_j \quad \text{- force per unit volume}$$

Conservation of momentum

momentum equation

***for an elementary volume* dV :**

$$\frac{\partial}{\partial t}\left(\rho u_j\right)+\frac{\partial}{\partial x_i}\left(\rho u_j u_i\right)=F_j$$

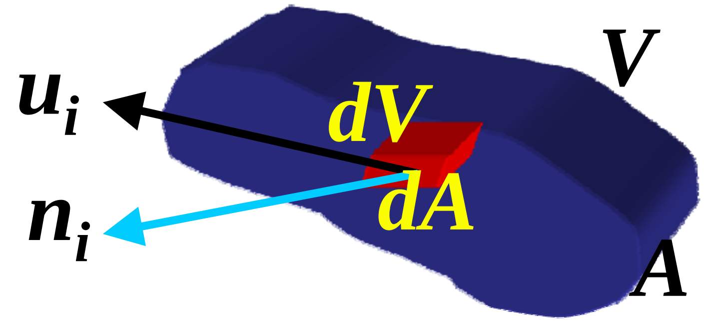
$$\frac{\partial}{\partial t}\left(\rho u_j\right)+\frac{\partial}{\partial x_i}\left(\rho u_j u_i\right)=\rho f_j+\frac{\partial \sigma_{ij}}{\partial x_i}$$

$$-\frac{\partial p}{\partial x_j}+\frac{\partial \sigma_{ij}^{d}}{\partial x_i}$$

Conservation of momentum

integrated for a final volume V

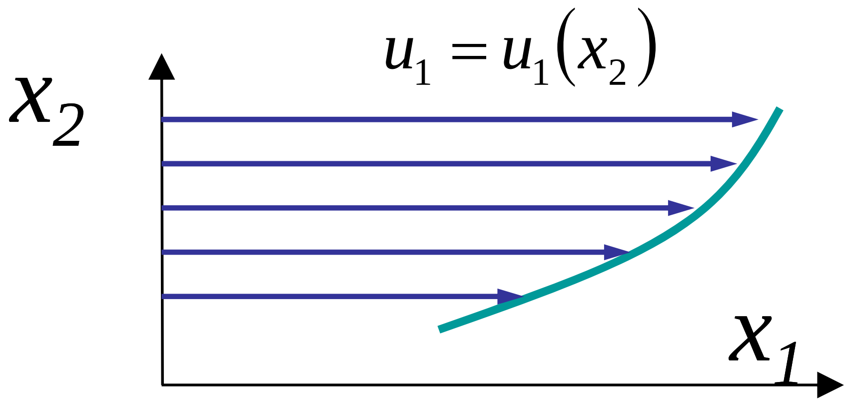
bounded by a surface A :

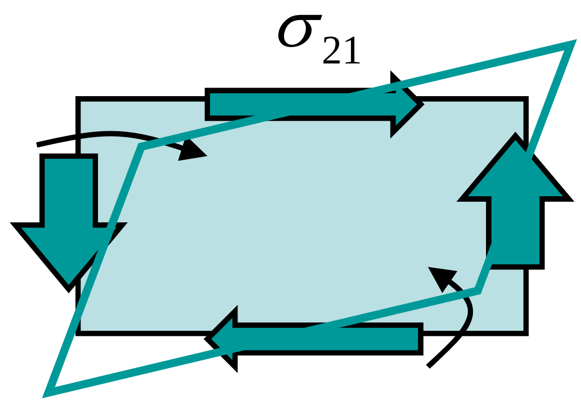


$$\overbrace{\int_V \frac{\partial}{\partial t}\left(\rho u_j\right) dV}^{\text{momentum change}} = \overbrace{-\int_A u_j \rho (n_i . u_i) dA}^{\substack{\text{convection} \\ \text{outlet}-\text{inlet}}}$$

$$+ \underbrace{\int_V \rho f_j dV}_{\text{body forces}} - \underbrace{\int_A p n_j dA}_{\text{spherical stress}} + \underbrace{\int_A \sigma_{ij}^d n_i dA}_{\text{deformation stress}}$$

Stress - strain relationship





$$dF_j = \frac{\partial \sigma_{ij}}{\partial x_i} dV = \left(-\delta_{ij} \frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}^d}{\partial x_i} \right) dV$$

$$u_2 = u_3 = 0 \quad \Rightarrow \quad \frac{\partial u_2}{\partial x_2} = \frac{\partial u_3}{\partial x_3} = 0$$

$$\Rightarrow \quad \frac{\partial u_1}{\partial x_1} = 0 \quad \Rightarrow \quad u_1 = u_1(x_2) \qquad \omega_{12} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} \right) = \frac{1}{2} \frac{\partial u_1}{\partial x_2}$$

Stress-strain relationship

for Newtonian fluids :

$$\sigma_{12} = \mu \frac{\partial u_1}{\partial x_2} = 2\mu\omega_{12}$$

generalized :

$$\sigma_{ij}^d = 2\mu\omega_{ij}^d$$

deformation stress = 2 X viscosity X deformation strain rate

$$\omega_{ij} = \frac{1}{3}\delta_{ij}\omega_{kk} + \omega_{ij}^d$$

$$\sigma_{ij}^d = 2\mu\left[\frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) - \frac{1}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right]$$

Navier-Stokes equations

$$\frac{\partial}{\partial t}\left(\rho u_j\right) + \frac{\partial}{\partial x_i}\left(\rho u_j u_i\right) = \rho f_j - \frac{\partial p}{\partial x_j} + \frac{\partial \sigma_{ij}^d}{\partial x_i}$$

$$\frac{\partial \sigma_{ij}^d}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\mu \frac{\partial u_j}{\partial x_i}\right) + \frac{\partial}{\partial x_i}\left(\mu \frac{\partial u_i}{\partial x_j}\right) - \frac{2}{3}\frac{\partial}{\partial x_i}\left(\mu \delta_{ij} \frac{\partial u_k}{\partial x_k}\right)$$

$$\frac{\partial \sigma_{ij}^d}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\mu \frac{\partial u_j}{\partial x_i}\right) + \frac{1}{3}\frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_i}{\partial x_i}\right)$$

Navier-Stokes equations

$$\frac{\partial}{\partial t}\left(\rho u_j\right)+\frac{\partial}{\partial x_i}\left(\rho u_j u_i\right)=$$

$$\rho f_j-\frac{\partial p}{\partial x_j}+\frac{\partial}{\partial x_i}\left(\mu\frac{\partial u_j}{\partial x_i}\right)+\frac{1}{3}\frac{\partial}{\partial x_j}\left(\mu\frac{\partial u_i}{\partial x_i}\right)$$

for an incompressible fluid $\rho = const$

with a constant viscosity $\mu = const$

$$\frac{\partial u_j}{\partial t}+\frac{\partial}{\partial x_i}\left(u_j u_i\right)= f_j-\frac{1}{\rho}\frac{\partial p}{\partial x_j}+\frac{\mu}{\rho}\frac{\partial^2 u_j}{\partial x_i^2}$$

Surface force decomposition

$$dF_j = \frac{\partial \sigma_{ij}}{\partial x_i} dV = \left(-\delta_{ij} \frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}^d}{\partial x_i} \right) dV$$

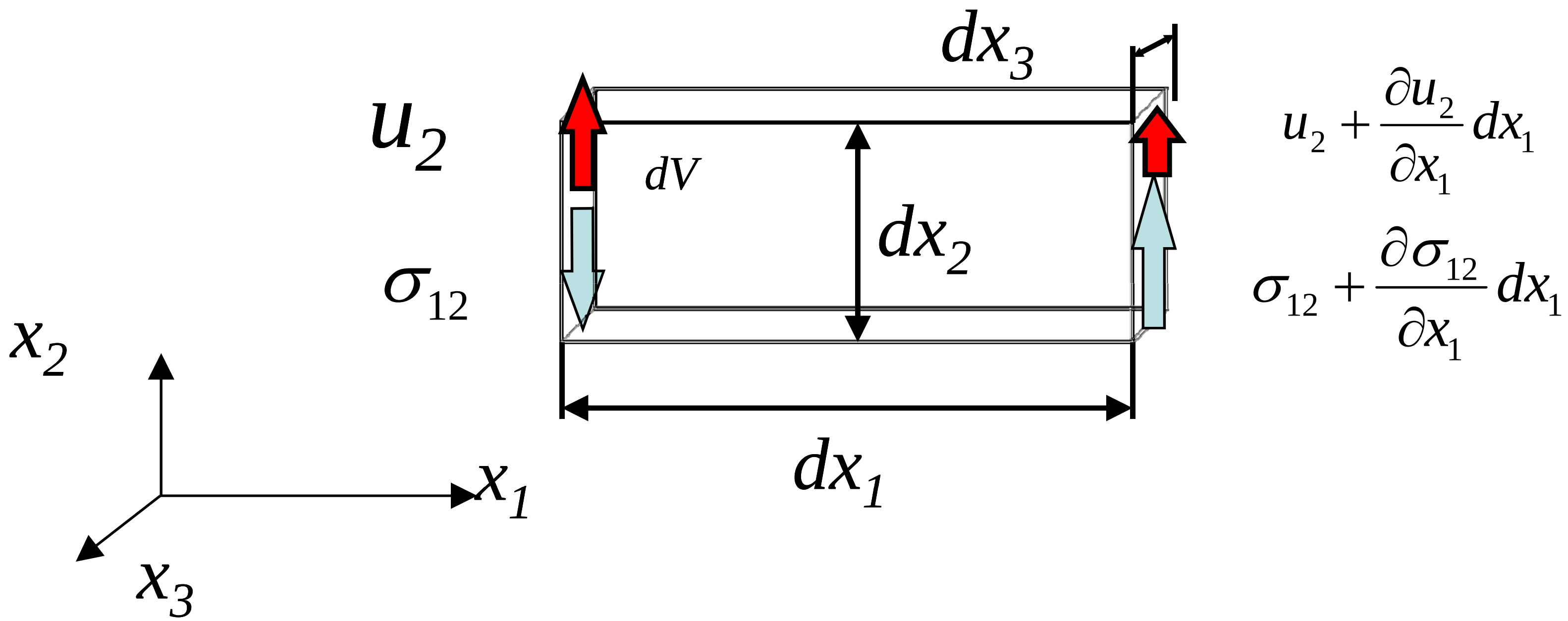
since:

$$\delta_{ij} \frac{\partial p}{\partial x_i} = \frac{\partial p}{\partial x_j}$$

the total stress F_j can be separated into the spherical and deformation components:

$$dF_j = -\frac{\partial p}{\partial x_j} dV + \frac{\partial \sigma_{ij}^d}{\partial x_i} dV$$

Work decomposition



total work done on an elementary volume:

$$\left(\sigma_{12} + \frac{\partial \sigma_{12}}{\partial x_1} dx_1\right) dx_2 dx_3 \left(u_2 + \frac{\partial u_2}{\partial x_1} dx_1\right)$$

$$-\sigma_{12} dx_2 dx_3 u_2 =$$

Work decomposition

$$= \left(u_2 \frac{\partial \sigma_{12}}{\partial x_1} + \sigma_{12} \frac{\partial u_2}{\partial x_1} \right) dx_1 dx_2 dx_3 = \frac{\partial}{\partial x_1} \left(u_2 \sigma_{12} \right) dV$$

generalized - total work done on an elementary volume can be separated into the kinetic and deformation components:

$$\underbrace{\frac{\partial}{\partial x_i} \left(u_j \sigma_{ij} \right)}_{\substack{\text{total} \\ \text{work}}} = \underbrace{u_j F_j}_{\substack{\text{kinetic} \\ \text{work}}} + \underbrace{\sigma_{ij} \frac{\partial u_j}{\partial x_i}}_{\substack{\text{deformation} \\ \text{work}}}$$

Conservation of mechanical energy

"change of kinetic energy is a result of work done by external forces"

start with the momentum equation :

$$\frac{D}{Dt}\left(\rho dV u_j\right) = F_j \qquad \Big/ \times u_j$$

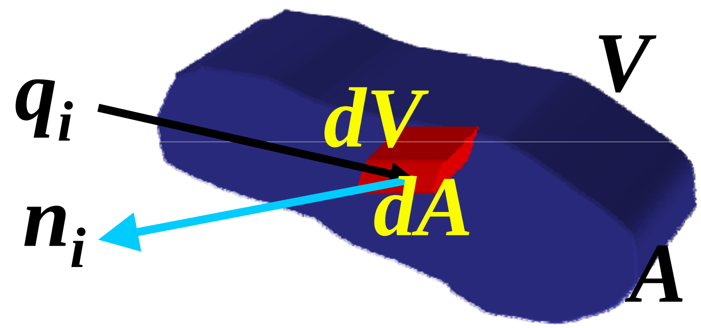
$$\underbrace{\frac{D}{Dt}\left(\rho dV \frac{u_j u_j}{2}\right)}_{\text{kinetic energy change}} = \underbrace{\left(f_j \rho dV + \frac{\partial \sigma_{ij}}{\partial x_i} dV\right) u_j}_{\text{work by external forces}}$$

Conservation of thermal (or internal) energy

Net heat flux entering a control volume results in rate of change of internal energy

$$\frac{D}{Dt}\left(\upsilon\rho dV\right) = -\frac{\partial q_i}{\partial x_i} + \sigma_{ij}\omega_{ij}$$

$$q_i = -k\frac{\partial T}{\partial x_i}$$



$$\underbrace{\frac{D}{Dt}\left(\upsilon\rho dV\right)}_{\text{internal energy change}} = \underbrace{\frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right)}_{\substack{\text{conduction} \\ \text{heat transfer}}} + \underbrace{\sigma_{ij}\frac{\partial u_j}{\partial x_i}}_{\substack{\text{deformation work} \\ \text{heat transfer}}}$$

Conservation of thermal energy

set :

$\varphi = cT$ - specific heat × temperature

$\phi = \int_V cT\rho dV$ - internal energy

$\psi = \frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right) + \sigma_{ij}\frac{\partial u_j}{\partial x_i}$ - heat transfer

thermal energy equation

for an elementary volume dV :

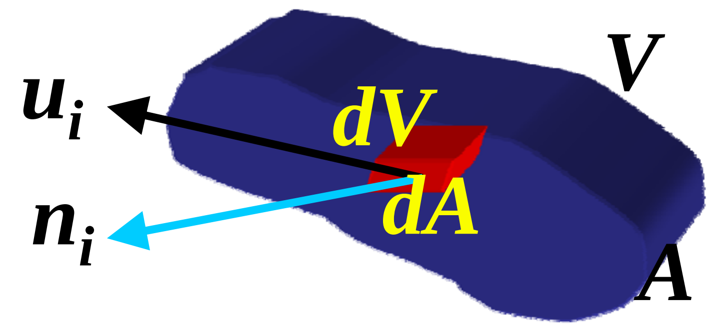
$$\frac{\partial}{\partial t}(\rho cT) + \frac{\partial}{\partial x_i}(\rho cTu_i) =$$

$$= \frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right) + \sigma_{ij}\frac{\partial u_j}{\partial x_i}$$

Conservation of thermal energy

integrated for a final volume V

bounded by a surface A :



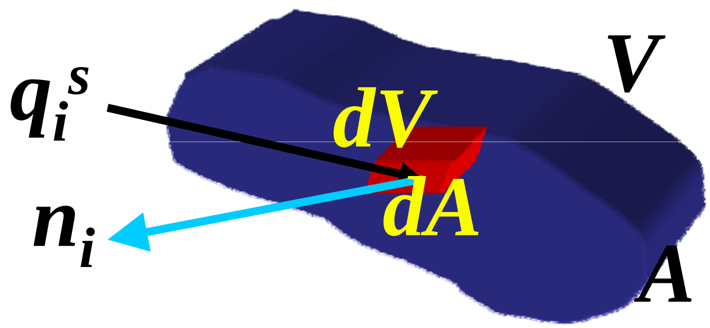
$$\overbrace{\int_V \frac{\partial}{\partial t}(\rho c T)\, dV}^{\text{internal energy change}} = \overbrace{-\int_A \rho c T n_i u_i \, dA}^{\substack{\text{internal energy convection} \\ \text{inlet – outlet}}}$$

$$+ \underbrace{\int_A k \frac{\partial T}{\partial x_i} n_i \, dA}_{\substack{\text{conduction} \\ \text{heat transfer}}} + \underbrace{\int_V \sigma_{ij} \frac{\partial u_j}{\partial x_i} \, dV}_{\substack{\text{deformation work} \\ \text{heat transfer}}}$$

Conservation of chemical species

Net species mass transfer across the control volume faces + chemical reaction within the control volume results in a change of concentration of chemical Species in the control volume

$$\frac{D}{Dt}\left(m^s \rho dV\right) = -\frac{\partial q_i^s}{\partial x_i} + R^s$$



$$q_i^s = -\mathbf{D}^s \frac{\partial m^s}{\partial x_i}$$

$$\underbrace{\frac{D}{Dt}\left(m^s \rho dV\right)}_{\text{concentration change}} = \underbrace{\frac{\partial}{\partial x_i}\left(\mathbf{D}^s \frac{\partial m^s}{\partial x_i}\right)}_{\text{diffusion mass transfer}} + \underbrace{R^s}_{\text{source/sink due to chemical reaction}}$$

Conservation of chemical species

set :

$\varphi = m^s$ - species mass concentration

$\phi = \int_V m^s \rho dV$ - species mass

$\psi = \frac{\partial}{\partial x_i}\left(\mathbf{D}^s \frac{\partial m^s}{\partial x_i} \right) + R^s$ - diffusion and chemical reaction

mass transfer equation

for an elementary volume dV :

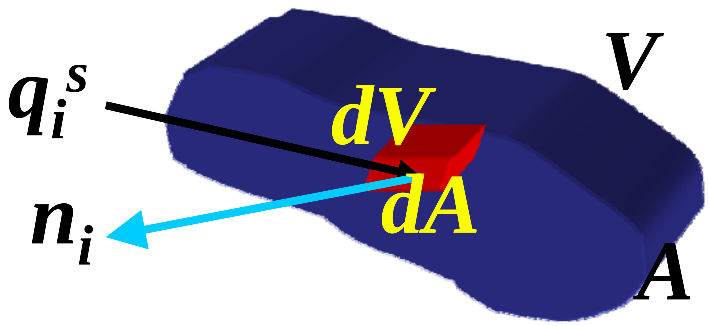
$$\frac{\partial}{\partial t}\left(\rho m^s\right) + \frac{\partial}{\partial x_i}\left(\rho m^s u_i\right) =$$

$$= \frac{\partial}{\partial x_i}\left(\mathbf{D}^s \frac{\partial m^s}{\partial x_i} \right) + R^s$$

Conservation of chemical species

integrated for a final volume V

bounded by a surface A :



$$\underbrace{\int_V \frac{\partial}{\partial t}\left(\rho m^s\right) dV}_{\text{species concentration change}} = \underbrace{-\int_A \rho m^s n_i u_i dA}_{\text{species convection inlet – outlet}}$$

$$+ \underbrace{\int_A \mathbf{D}^s \frac{\partial m^s}{\partial x_i} n_i dA}_{\text{species diffusion mass transfer}} + \underbrace{\int_V R^s dV}_{\text{source/sink due to chemical reaction}}$$

Class of Linear Second-order PDEs

- Linear second-order PDEs are of the form

$$Au_{xx} + 2Bu_{xy} + Cu_{yy} + Eu_x + Fu_y + Gu = H$$

where A - H are functions of x and y only

- Elliptic PDEs: $B^2 - AC < 0$

  (steady state heat equations without heat source)

- Parabolic PDEs: $B^2 - AC = 0$

  (transient heat transfer equations)

- Hyperbolic PDEs: $B^2 - AC > 0$

  (wave equations)

MODULE 1: Review Questions

- What is the driving force for (a) heat transfer (b) electric current flow and (c) fluid flow?
- Which one of the following is not a material property?

  (a) thermal conductivity (b) heat transfer coefficient (c)emissivity
- What is the order of magnitude of thermal conductivity for (a) metals (b) solid insulating materials (c) liquids (d) gases?
- What are the orders of magnitude for free convection heat transfer coefficient, forced convection and boiling?
- Under what circumstances can one expect radiation heat transfer to be significant?
- An ideal gas is heated from 40ºC to 60ºC (a) at constant volume and (b) at constant pressure. For which case do you think the energy required will be greater? Explain why?
- A person claims that heat cannot be transferred in a vacuum. Evaluate this claim.

MODULE 1: Review Questions (contd...)

- In which of the three states (solid/liquid/gas) of a matter, conduction heat transfer is high/low? Explain your claim.
- Name some good and some poor conductors of heat.
- Show that heat flow lines and isotherms in conduction heat transfer are normal to each other. Will this condition hold for convection heat transfer?
- Distinguish between Eulerian and Lagrangian approach in fluid mechanics. Why is the Eulerian approach normally used?
- Derive the Reynolds transport equation.
- Derive the continuity (mass conservation) equation in differential form for incompressible flow.
- Define substantial derivative. What is the physical significance of the substantial derivative in an Eulerian framework?