

## LECTURE 12 TO 14 – HYDRAULIC ACTUATORS

### SELF EVALUATION QUESTIONS AND ANSWERS

1A pump supplies oil at  $0.002 \text{ m}^3/\text{s}$  to a 50mm diameter double acting cylinder and a rod diameter is 20mm. If the load is 6000N both in extending and retracting, find

- Piston velocity during the extension stroke and retraction stroke
- Pressure during the extension stroke and retraction stroke
- Power during the extension stroke and retraction stroke

2A hydraulic cylinder has to move a table of weight 13kN. Speed of the cylinder is to be accelerated up to a velocity of 0.13m/s in 0.5 seconds and brought to stop within a distance of 0.02m. Assume coefficient of sliding friction as 0.15 and cylinder bore diameter as 50mm. Calculate the surge pressure.

3.A cylinder has a bore of 80mm diameter and a rod of 45mm diameter. It drives a load of 7000N, travelling at a velocity of 15m/min. The load slides on a flat horizontal surface having a coefficient of friction of 0.12. The load is to be decelerated to rest within a cushion length of 20mm. If the relief valve is set at 50 bar, compute the fluid pressure developed in the cushion.

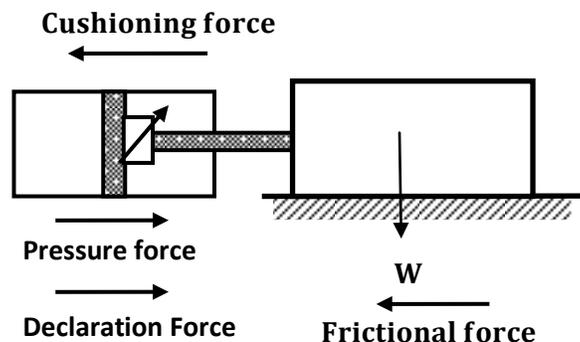


Figure 1 for the Problem No 3

4. A cylinder has a bore of 125mm diameter and a rod of 70mm diameter. It drives a load of 2000 kg vertically up and down at a maximum velocity of 3 m/s. The load is slowed down to rest in the cushion length of 50mm. If the relief valve is set at 140 bar, determine the average pressure in the cushions while extending and retracting.

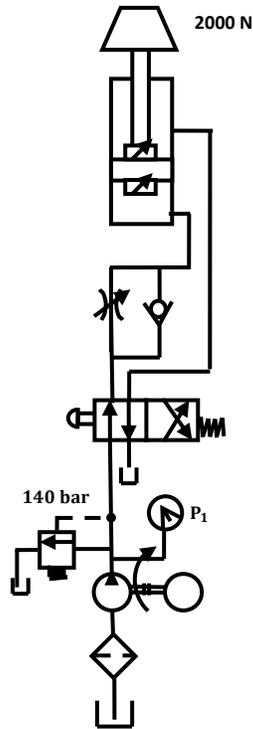


Figure 2 for the Problem No 4

5 For a second class lever system given in figure 3 determine the hydraulic cylinder force required to overcome the load.

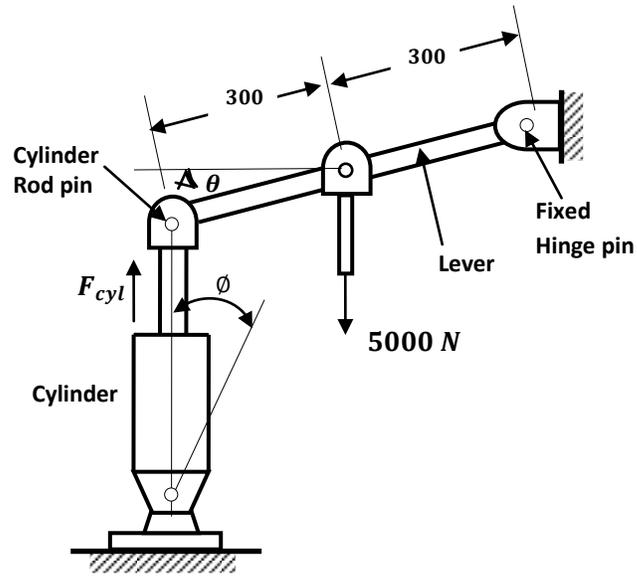


Figure 3 for the Problem No 5

6 For the crane system given in figure below, find the cylinder the force required to lift a load of 9000 N.

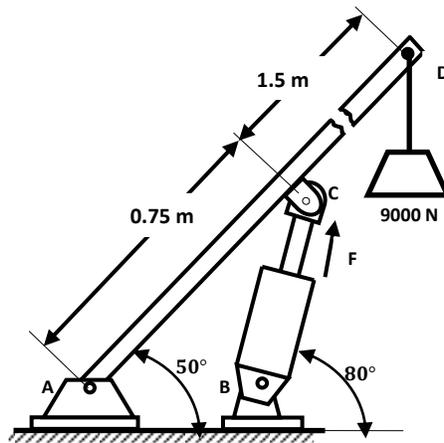


Figure 4 for the Problem No 6

7. For a bent lever system shown in figure 5, the cylinder force is 1400N. How much load can the system drive.

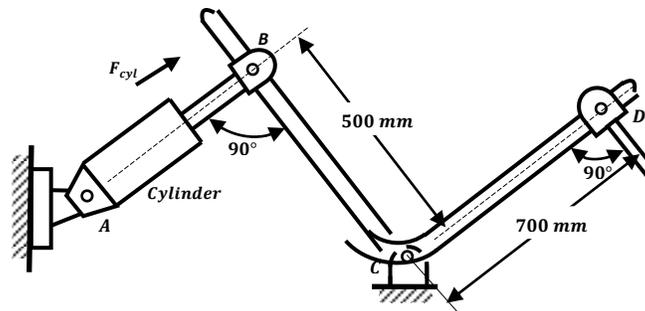


Figure 5 for the Problem No 7

8: A two stage telescopic cylinder is used to tilt the body of a lorry. When the lorry is fully laden, the cylinder has to exert a force equivalent to 40kN at all points in its stroke. The outside diameters of the tubes forming two stages are 75mm and 100mm. If the pump powering the cylinder delivers 12 l/min, calculate the extend speed and pressure required for each stage of the cylinder when tilting fully laden lorry.

9: A mass of 2000kg is to be accelerated upto a velocity of 1m/s from rest over a distance of 50mm. The coefficient of friction between the load and the guides is 0.15. Select the bore of the cylinder required to accelerate this load if the maximum allowable pressure at the full bore end is 100 bar. Take seal friction to be equivalent to a pressure drop of 5 bar

10. A Press cylinder has to exert a force to lift the crosshead and tooling. The cylinder is rigidly fixed by a front flange and the load is pivoted and fully guided. Maximum thrust from the press is 20kN with a stroke of 1.7m. Determine the suitable cylinder. The system working pressure should not exceed 250 bar.

11. A hydraulic cylinder has to move a load horizontally through a distance of 3m. The cylinder is front flange mounted and load is rigidly connected to the piston rod and fully guided. The extend force to be exerted by the cylinder is 1.6 tonnes and the retract force is 0.7 tonnes. Assume the effective dynamic thrust is 0.9 tonnes the static thrust,

If the system pressure is limited to 150 bar

Determine a suitable standard metric size of cylinder

Calculate the actual operating pressure

12. A machine tool cylinder is connected regeneratively to give a rapid approach speed of 10 m/min for a stroke of 1 m with a theoretical thrust of 2.5 Tonnes. It is then switched to conventional connection to provide a pressing speed of 0.25 m/min for 0.5 m with a theoretical thrust of 10 Tonnes. The maximum pressure at the cylinder is to be 200 bar.

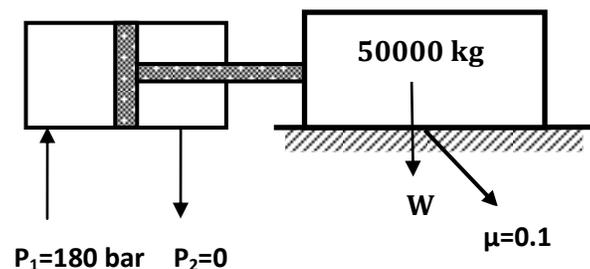
Select a suitable standard metric cylinder

Calculate the pump delivery required for both parts of extend stroke.

13 A hydraulic cylinder is to accelerate a load of 50 Tonnes horizontally from rest with a velocity of 10 m/min to 50mm . Take coefficient of friction between the load and the guide as 0.1. Assume zero back pressure. Determine

a) a suitable size of standard metric cylinder if the maximum allowable pressure at the cylinder is 180 bar

b) the fluid flow rate required to drive the piston forward at 3m/min



### Q1Solution

Oil flow rate from pump,  $Q = 0.002 \text{ m}^3/\text{s}$

Diameter of the cylinder,  $D = 50\text{mm}$

$$= 0.05 \text{ m}$$

Diameter of the rod,  $d = 20\text{mm}$

$$= 0.02\text{m}$$

Load during the extension and retraction  $F = 6000\text{N}$

a. Piston velocity during extension stroke  $V_E = \frac{Q}{A_P}$

$$= \frac{0.002}{\frac{\pi}{4} \times 0.05^2}$$

$$= \mathbf{1 \text{ m/s}}$$

Piston velocity during retraction stroke  $V_R = \frac{Q}{A_P - A_R}$

$$= \frac{0.002}{\frac{\pi}{4} \times (0.05^2 - 0.02^2)} = 1.2 \text{ m/s}$$

Cylinder pressure during extension stroke  $P_E = \frac{F}{A_P} = \frac{6000}{\frac{\pi}{4} \times 0.05^2} = 30.6 \text{ bar}$

Cylinder pressure during retraction stroke  $P_R = \frac{F}{A_P - A_R} = \frac{6000}{\frac{\pi}{4} \times (0.05^2 - 0.02^2)} = 36.4 \text{ bar}$

Cylinder power during extension stroke  $= \frac{P_E \times Q}{1000} = \frac{30.6 \times 10^5 \times 0.002}{1000} = 6.12 \text{ kW}$

Cylinder power during extension stroke  $= \frac{P_R \times Q}{1000} = \frac{36.4 \times 10^5 \times 0.002}{1000} = 7.28 \text{ kW}$

## Q2 Solution

Initial velocity  $u = 0\text{m/s}$

Final velocity  $v = 0.13\text{m/s}$

$$\text{Acceleration } a = \frac{v-u}{t} = \frac{0.13-0}{0.5} = \mathbf{0.26 \text{ m/s}^2}$$

Force required to move the piston = Dynamic force + frictional force

$$\begin{aligned} &= \left[ \frac{w}{g} \times a \right] + \mu \cdot w = \left[ \frac{13000}{9.81} \times 0.26 \right] + 0.15 \times 13000 \\ &= \mathbf{2294.5\text{N}} \end{aligned}$$

To overcome this force, the pressure required in the hydraulic cylinder is

$$= \frac{2294.5}{\frac{\pi}{4} \times 0.05^2} = 11.69 \text{ bar}$$

From the equation for velocity, acceleration and distance  $v^2 - u^2 = 2as$

$$a = \frac{v^2 - u^2}{2s} = \frac{0^2 - 0.13^2}{2 \times 0.02} = -0.4225\text{m}$$

**(The -ve sign indicates that it is deceleration)**

The total force required to stop the motion of a cylinder

$$= \frac{13000}{9.81} \times 0.4225 + 13000 \times 0.15 = 2510\text{N}$$

Then pressure created by this opposing force is

$$= \frac{2510}{\frac{\pi}{4} \times 0.05^2} = 12.78 \text{ bar}$$

Thus surge pressure  $P_s = P_1 + P_2 = 11.69 + 12.78 = \mathbf{24.47 \text{ bar}}$

### Q3 Solution

Cushion length  $s = 20\text{mm} = 0.02\text{m}$

Velocity  $u = 15 \text{ m/min} = 0.25 \text{ m/s}$

From the equation of motion,

$$v^2 = u^2 + 2as \text{ (final velocity is zero)}$$

$$a = \frac{-u^2}{2s}$$

$$\begin{aligned} \text{Decelerating force to retard load} &= \frac{w}{g} \times a = \frac{w}{g} \times \frac{u^2}{2s} = \frac{6700 \times 0.25^2}{9.81 \times 2 \times 0.02} \\ &= \mathbf{1067\text{N}} \end{aligned}$$

$$\begin{aligned} \text{Pressure force on blank end} &= P \times A = 50 \times 10^5 \times \frac{\pi}{4} \times 0.08^2 \\ &= \mathbf{25133\text{N}} \end{aligned}$$

Friction force  $= \mu \cdot w$

$$= 0.12 \times 6700$$

$$= \mathbf{804 \text{ N}}$$

Cushion force  $= (\text{Pressure force} + \text{Decelerating force}) - \text{Friction force}$

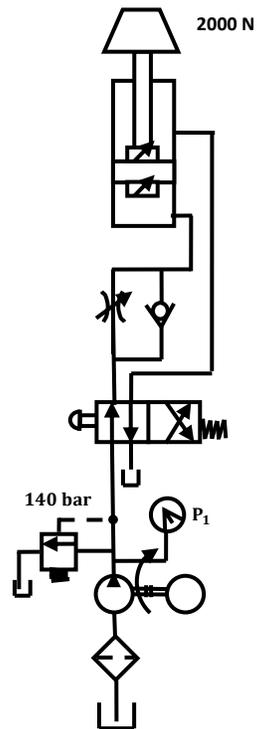
$$= 25133 + 1067 - 804$$

$$= \mathbf{25396\text{N}}$$

$$\begin{aligned} \text{Fluid pressure developed at the cushion} &= \frac{F}{A_P - A_R} \\ &= \frac{25396}{\frac{\pi}{4}(0.08^2 - 0.045^2)} \end{aligned}$$

$$= \mathbf{74 \text{ bar}}$$

## Q4 Solution



From the equation of motion,

$$v^2 = u^2 + 2as \text{ (final velocity is zero)}$$

$$a = \frac{-u^2}{2s}$$

∴ Decelerating force to retard = m.a

$$= \frac{2000 \times 3^2}{2 \times 0.05}$$

$$= \mathbf{180 \text{ kN}}$$

Weight of the load = mg

$$= 2000 \times 9.81 = 19.6 \text{ Kn}$$

a. During extension

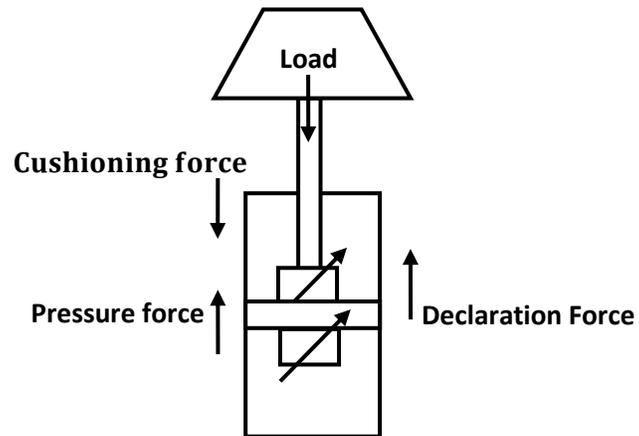


Figure E8.15 for the Problem No E4

$$\text{Pressure force} = P \times A_p$$

$$= 140 \times 10^5 \times \frac{\pi}{4} \times 0.125^2$$

$$= \mathbf{171.8 \text{ kN}}$$

$$\text{Cushion force} = (\text{Pressure force} + \text{Decelerating force} - \text{weight})$$

$$= 171.8 + 180 - 19.6$$

$$= \mathbf{332.2 \text{ kN}}$$

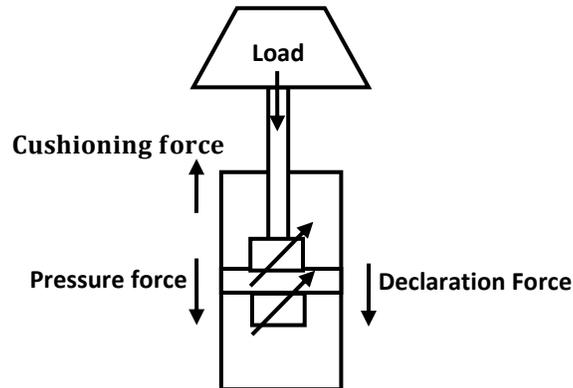
$$\text{Cushion pressure} = \frac{\text{cushion force}}{A_p - A_R}$$

$$= \frac{332.2}{\frac{\pi}{4}(0.125^2 - 0.07^2)}$$

$$= 39438 \text{ kN/m}$$

$$= \mathbf{394.38 \text{ bar}}$$

**b. During retraction**



**Figure E8.16 for the Problem No E4**

$$\text{Pressure force} = P (A_P - A_R)$$

$$= 140 \times 10^5 \times \frac{\pi}{4} (0.125^2 - 0.07^2)$$

$$= \mathbf{117.9 \text{ kN}}$$

$$\text{Cushion force} = (\text{Pressure force} + \text{Decelerating force} + \text{weight})$$

$$= 171.8 + 180 + 19.6$$

$$= \mathbf{317.5 \text{ kN}}$$

$$\text{Cushion pressure} = \frac{\text{cushion force}}{A_P}$$

$$= \frac{317.5}{\frac{\pi}{4} (0.125^2)}$$

$$= 25872 \text{ kN/m}^2$$

$$= \mathbf{258.72 \text{ bar}}$$

### Q5 Solution

Taking moments about C,  $F_c \times (L_1 + L_2) \cos \theta = W \times L_2 \cos \theta$

$$\begin{aligned}\text{Cylinder Force } F_c &= W \times \frac{L_2}{L_1 - L_2} \\ &= 5000 \times \frac{0.3}{0.3 + 0.3} \\ &= \mathbf{2500 \text{ N}}\end{aligned}$$

### Q6 Solution

$F_c$  is the cylinder force

Taking moments about A,

$$W \times (1.5 + 0.75) \times \cos 50 + F_c \times \cos 80 (0.75 \sin 50) = F_c \sin 80 (0.75 \cos 50)$$

$$F_c = \frac{9000 \times 2.25 \cos 50}{0.75 \times \sin 30} = 34710 \text{ N}$$

### Q7 Solution

Taking moments about O,  $F_c \times 0.5 = W \times 0.7$

$$\begin{aligned}\text{Load } W &= \frac{F_c \times 0.5}{0.7} \\ &= \frac{1400 \times 0.5}{0.7} \\ &= \mathbf{1000 \text{ N}}\end{aligned}$$

### Q8 Solution

**First stage**

$$\text{First stage speed} = \frac{Q}{A_1} = \frac{12 \times 10^{-3}}{\frac{\pi}{4} \times 0.1^2} = \mathbf{1.52 \text{ m/min}}$$

$$\text{First stage pressure} = \frac{F}{A_1} = \frac{40 \times 10^3}{\frac{\pi}{4} \times 0.1^2} = \mathbf{50.9 \text{ bar}}$$

## Second stage

$$\text{Second stage speed} = \frac{Q}{A_2} = \frac{12 \times 10^{-3}}{\frac{\pi}{4} \times 0.075^2} = \mathbf{2.72 \text{ m/min}}$$

$$\text{Second stage pressure} = \frac{F}{A_2} = \frac{40 \times 10^3}{\frac{\pi}{4} \times 0.075^2} = \mathbf{90.5 \text{ bar}}$$

## Q9 Solution

In this case

$$u=0$$

$$v=1\text{m/s}$$

$$s = 0.05\text{m}$$

Using the equation,

$$v^2 = u^2 + 2as$$

$$a = \frac{v^2 - u^2}{2s}$$

$$= \frac{1 - 0}{2 \times 0.05}$$

$$= 10\text{m/s}$$

Force to accelerate this load  $F = m \cdot a$

$$= 2000 \times 10$$

$$= 20000\text{N}$$

Force to overcome load friction  $P = \mu \cdot W$

$$= 0.15 \times 2000 \times 9.81$$

$$= 2943\text{N}$$

$$\text{Total force} = F + P$$

$$= 20000 + 2943$$

$$= \mathbf{22943\text{N}}$$

Pressure available = (pressure at full bore end) - (pressure drop due to seal friction)

$$= 100 - 5$$

$$= 95 \text{ bar}$$

$$= \mathbf{95 \times 10^5 \text{ N/m}^2}$$

Cylinder area required for given thrust

$$A = \frac{\text{total force}}{\text{pressure}}$$

$$= \frac{22943}{95 \times 10^5}$$

$$= 0.002415 \text{ m}^2$$

Diameter of cylinder bore

$$D = \left( \frac{4 \times A}{\pi} \right)^{1/2} = \left( \frac{4 \times 0.002415}{\pi} \right)^{1/2} = 0.0554 \text{ m}$$

$$= \mathbf{55.4 \text{ mm}}$$

The nearest standard cylinder has a 63mm diameter bore.

## Q10 Solution

### Piston rod diameter

Load  $F = 200\text{kN}$

Modulus of Elasticity  $E = 2.1 \times 10^{11} \text{ N/m}^2$

Equivalent buckling length  $L = \frac{\text{cylinder stroke}}{\sqrt{2}}$

$$= \frac{1.7}{\sqrt{2}}$$

$$= 1.2\text{m}$$

Consider the factor of safety (S) as 3.5

Buckling load  $K = F \times S$

$$= 200 \times 10^3 \times 3.5$$

$$= 700 \times 10^3 \text{ N}$$

Buckling load  $K = \frac{\pi^2 EI}{L^2}$  where  $I = \frac{\pi d^4}{64}$

$$= \frac{\pi^2 E}{L^2} \times \frac{\pi d^4}{64}$$

$$\therefore d^4 = K \left( \frac{L^2}{\pi^2 \times E} \right) \left( \frac{\pi d^4}{64} \right)$$

$$= 700 \times 10^3 \times \left( \frac{1.2^2}{\pi^2 \times 2.1 \times 10^{11}} \right) \times \left( \frac{64}{\pi} \right)$$

$$= 992 \times 10^{-8} \text{ m}^4$$

Piston rod diameter  $d = (992 \times 10^{-8})^{1/4}$

$$= 0.056\text{m}$$

$$= 56\text{mm}$$

This is of standard size ( from table)

Check for bore diameter :-

Maximum allowable pressure = 250 bar

$$= 250 \times 10^5 \text{ N/m}^2$$

Assume dynamic thrust = 0.9 x pressure x area

$$\begin{aligned} \text{Piston area } A &= \frac{\text{thrust}}{0.9 \times \text{pressure}} \\ &= \frac{200 \times 10^3}{0.9 \times 250 \times 10^5} \\ &= 0.0088 \text{ m}^2 \end{aligned}$$

$$\text{Bore diameter } D = \left[ \frac{4 \times A}{\pi} \right]^{1/2}$$

$$= \left[ \frac{4 \times 0.0088}{\pi} \right]^{1/2}$$

$$= 0.105 \text{ m}$$

$$= 105\text{mm}$$

But from the table of preferred sizes suitable bore diameter is 125 mm and corresponding piston rod diameter is 70mm. This diameter is more than the previously calculated value. So it can withstand the buckling load.

Thus, Bore diameter = 105mm

Diameter of piston rod = 70mm

## Preferred Sizes

BS : 5785 1980 gives a table of the preferred sizes for the cylinder bore and rod diameter. Most manufacturers are offering two rod sizes for each cylinder bore size. It is shown in following table

Piston diameter(mm)		40	50	63	80	100	125	140
Piston rod Diameter(mm)	Small	20	28	36	45	56	70	90
	Large	28	36	45	56	70	90	100

Piston diameter(mm)		160	180	200	220	250	280	320
Piston rod Diameter(mm)	Small	100	110	125	140	160	180	200
	Large	110	125	140	160	180	200	220

The large piston rod to piston diameter ratio is in the range of 0.7, which gives an annulus area which is approximately one half the full bore area. This ratio is used in regenerative circuits to give similar values of speed and thrust on both the extending and retracting strokes.

### **Q11 Solution**

a. Piston rod diameter

The first step in the solution is to determine the minimum piston rod diameter for the buckling strength.

The thrust force exerted by the cylinder = 1.6 Tonnes =  $1.6 \times 1000 \times 9.81 = 15696 \text{ N}$

Assuming a factor of safety of 3.5

Bucking load  $K = 3.5 \times 15696 = 54936 \text{ N}$

The cylinder is front flange mounted and load rigidly connected to the piston rod and fully guided. For these conditions

Free buckling length  $L = \frac{l}{2} = \frac{\text{stroke}}{2} = \frac{3.2}{2} = 1.5 \text{ m}$

Buckling load  $K = \frac{\pi^2 EI}{L^2}$  where  $I = \frac{\pi d^4}{64}$

$$= \frac{\pi^2 E}{L^2} \times \frac{\pi d^4}{64}$$

$$\therefore d^4 = K \left( \frac{L^2}{\pi^2 \times E} \right) \left( \frac{\pi d^4}{64} \right)$$

$$= 54936 \times \left( \frac{1.5^2}{\pi^2 \times 2.1 \times 10^{11}} \right) \times \left( \frac{64}{\pi} \right)$$

Piston rod diameter  $d = 0.033 \text{ m} = \mathbf{33 \text{ mm}}$

Select the standard size of 36mm from the above table.

Check for bore diameter :-

Maximum allowable pressure = 250 bar

$$= 250 \times 10^5 \text{ N/m}^2$$

Assume dynamic thrust = 0.9 x pressure x area

$$\text{Piston area } A = \frac{\text{thrust}}{0.9 \times \text{pressure}}$$

$$= \frac{200 \times 10^3}{0.9 \times 250 \times 10^5}$$

$$= \mathbf{0.001162 \text{ m}^2}$$

$$\text{Bore diameter } D = \left[ \frac{4 \times A}{\pi} \right]^{1/2}$$

$$= \left[ \frac{4 \times 0.001162}{\pi} \right]^{1/2}$$

$$= 0.038 \text{ m}$$

$$= \mathbf{38 \text{ mm}}$$

But from the table of preferred sizes suitable bore diameter is 50 mm and corresponding piston rod diameter is 36 mm.

So the 50mm bore diameter x 36 mm rod diameter cylinder is selected.

b) the extend pressure required to give a dynamic thrust of 15696 N

$$= \left( \frac{\text{thrust}}{\text{area}} \right) \times \frac{1}{0.9} = \left( \frac{15696}{\frac{\pi}{4} (0.05)^2} \right) \times \frac{1}{0.9} = 89 \times 10^5 \frac{\text{N}}{\text{m}^2} = 89 \text{ bar}$$

The retract pressure required to give a dynamic thrust of 6867 N

$$= \left( \frac{\text{thrust}}{\text{Annulus area}} \right) \times \frac{1}{0.9} = \left( \frac{15696}{\frac{\pi}{4} (0.05)^2 - \frac{\pi}{4} (0.036)^2} \right) \times \frac{1}{0.9} = 81 \times 10^5 \frac{\text{N}}{\text{m}^2} = 81 \text{ bar}$$

### Q12 Solution

During regeneration, the forward thrust is the difference between the forces on the full bore and annulus sides of piston.

Forward thrust =

### Solution

$$Q_{in} = 0.016 \text{ m}^3/\text{s}$$

$$F_{ext} = F_{ret} = 5000 \text{ N}$$

$$d_c = 40 \text{ mm} = 0.04 \text{ m}$$

$$d_r = 20 \text{ mm} = 0.02 \text{ m}$$

(a) Hydraulic pressure during the extending stroke: -

$$\text{Forward thrust} = P \frac{\pi D^2}{4} - P \frac{\pi (D^2 - d^2)}{4} = P \frac{\pi d^2}{4}$$

$$\text{Piston rod diameter} = \left( \frac{4 \times \text{Thrust}}{\pi \times P} \right)^{0.5} = \left( \frac{4 \times 2500 \times 9.81}{\pi \times 200 \times 10^5} \right)^{0.5} = 0.039 \text{ m} = 39 \text{ mm}$$

Select the standard rod diameter 45mm

During conventional connection

$$Thrust = P \frac{\pi D^2}{4}$$

$$Piston\ rod\ diameter = \left( \frac{4 \times Thrust}{\pi \times P} \right)^{0.5} = \left( \frac{4 \times 10000 \times 9.81}{\pi \times 200 \times 10^5} \right)^{0.5} 0.079m = 79mm$$

Select the standard piston diameter =80mm

So the bore diameter 80 x 45mm rod diameter cylinder is selected.

B. Rapid approach flow during

$$Regeneration = \frac{\pi d^2}{4} \times V_1 = \frac{\pi 0.0045^2}{4} \times 10 = 0.0159 \frac{m^3}{min} = 15.9 LPM$$

Flow during conventional connection

$$\frac{\pi D^2}{4} \times V_2 = \frac{\pi 0.0080^2}{4} \times 0.25 = 0.00126 \frac{m^3}{min} = 1.26 LPM$$

### Q13 Solution

In this case  $u=0$ ,  $v = 10m/min = 10/60 m/sec$

$s=0.05m$  and  $a$  is unknown.

Using the equation

$$v^2 = u^2 + 2as$$

$$0.16667^2 = 0^2 + 2ax0.05$$

$$a = 0.278 m/s^2$$

Force to accelerate load is given by

$$F = \left( \frac{W}{g} \right) a$$

$$F = \left( \frac{50000 \times 9.81}{9.81} \right) 0.278$$

$$F = 13900 N$$

Force P to overcome load friction is given by

$$P = \mu W$$

$$P = 0.15 \times 50000 \times 9.81 = 49050N$$

Total force to accelerate load and overcome friction is  $(F+P) = 13900+49050=62950$  N

Cylinder area required for a given thrust is calculated from

Thrust = Force x area

The Pressure available if pressure at full bore end of the cylinder less the equivalent seal break out pressure.

$$\text{Pressure available} = 189-0 = 180 \text{ bar} = 180 \times 10^5 \text{ N/mm}^2$$

$$\text{Area} = \frac{62950}{180 \times 10^5} = 0.003497 \text{ m}^2 = \frac{\pi D^2}{4}$$

Where D is the diameter

Substituting we get  $D = 67$  mm.

The nearest standard cylinder above has a 80 mm diameter bore.

**b) Flow rate required to drive the piston forward at 3 m/min**

$$\text{flow rate} = \text{area} \times \text{velocity} = \frac{\pi}{4} (0.080)^2 \times 3 = 0.0151 \frac{\text{m}^3}{\text{min}} = 15.1 \text{ LPM}$$