National Program on Technology Enhanced Learning

IIT MADRAS

Design and Optimization of Energy Systems

Final Examination

Duration: 3hr

Max. Marks: 100

1. Make suitable assumptions wherever required with justification
2. Assume any missing data
3. Graph sheet can be used for the Linear Programming problem

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(1) The dynamic viscosity of liquid water in Ns/m^2 varies with temperature (T in K) as given below.

S. No	**Temperature, K**	**Viscosity, Ns/m^2**
1	275	16.52×10^{-4}
2	290	10.8×10^{-4}
3	300	8.55×10^{-4}
4	315	6.31×10^{-4}

(a) Using Newton's divided difference method, appropriate for 4 data points, obtain en exact fit to the viscosity as a function of temperature. (6)
(b) Using the fit obtained in (a) estimate the viscosity at 295 K (1)
(c) Compare the result obtained in (b) with a linear interpolation. (1)

(2) An experiment was designed to examine the effect of load, x (in appropriate units) on the probability of failure of specimens of a certain industrial component. The following results were obtained from the experiment.

Load, x	Number of specimens	Number of failures
10	400	17
30	500	49
60	450	112
85	600	275
95	550	310
110	350	253

The regression model suitable for this problem is of the following form (also known as the logistic regression model)

$$p = \frac{1}{1+e^{-(a+bx)}}$$

Where "p" is the probability of failure of the component. Using the above model with the data given in the Table given above, get the "best" estimates of a and b. (10)

(b) Estimate the standard error of the estimate of "p" from the regression equation and the correlation coefficient. (2)

(3) The mass balance of three species (x_1, x_2 and x_3 all in kg/s) in a series of interconnected chemical reactors is given by the following equations:

$$1.4\,x_1 + 2.2\,x_2 + 4.8\,x_3 = 4.8$$
$$6\,x_1 - 3\,x_2 - 2x_3 = 22$$
$$2\,x_1 + 4\,x_2 + x_3 = 24$$

Using the Gauss-Seidel method with initial guess values of 1.0 for all the three variables, determine the values of x_1, x_2 and x_3. Perform at least 7 iterations and report the sum of the squares of the residues of the three variables at the end of every iteration. (10)

(4) (a) A closed rectangular metal box is to be constructed in order to maximize the volume. The total sheet metal available 24 m^2. Using the method of Lagrange multipliers for a constrained optimization in three variables (x_1, x_2 and x_3- the three dimensions of the box), determine the optimum dimensions of the box that maximize the volume. (10)

(b)For the above problem, report the optimal volume and the Lagrange multiplier. Establish using second order necessary and sufficient conditions that the optimum is indeed a maximum. (1+1+2)

(5) (a)Write down the Kuhn-Tucker conditions for the following Non-Linear optimization problem (NLP).

Minimize: $y = x_1^2+2x_2^2$
Subject to : $2x_1-2x_2=1$
$x_1^2+x_2^2 \le 4$ (4)

(b) Solve the above problem and check whether the inequality constraint is binding. (4)

(6) Newly harvested grain often has a high moisture content. This must be removed by drying to prevent spoilage and increase shelf life. The drying is usually achieved by warming ambient air and blowing it through a bed of the grain. The seasonal operating cost (in rupees) per square meter of grain bed for such a dryer consists of the heating cost plus the blower cost which are given by

Heating cost: Rs. $0.09\ Q\ \Delta T$

Blower cost: Rs. $1.1 \times 10^{-7}\ Q^3$

Where Q is the air flow rate though the bed during the season in $m^3/\ m^2$ of bed area and ΔT is the rise in temperature through the heater in ºC.

The values of Q and ΔT also influence the time required for adequate drying of the grain according to the following equation

Drying time = $70 \times 10^6 / Q^2 \Delta T$ days.

(a)Convert this into a single variable unconstrained optimization problem in air flow rate, Q for a drying time of exactly 50 days. (2)

(b) We would like to use GA to solve the above problem, as a single variable minimization problem for the total cost in terms of the air flow rate Q. Perform two iterations of the GA with an initial population size of 4. You may assume that $50 \leq Q \leq 1023\ m^3/m^2$. An accuracy of $1\ m^3/m^2$ is sufficient to represent Q. Decide appropriate strategies for crossover. No mutation is required. Report maximum, minimum and average fitness values for the initial population (which may be randomly chosen) and the populations at the end of the first and second iterations. There is no need to convert the minimization problem to a maximization problem. You may decide on the mating pool by giving more preference to those candidates which yield lower total coast. (14)

(7)Consider the problem of minimization of convective heat loss from a cylindrical storage heater that makes use of the solar energy collected by a suitable system. The volume of the tank is $5\ m^3$ and is fixed. The radius of the tank is “r” and the height is “h”.

(a) Convert this to a single variable unconstrained optimization problem in radius “r”. (1)

(b) We would like to use Simulated Annealing (SA) to solve the above problem, as a single variable minimization problem in “r”. Perform four iterations of the SA with a starting value of r = 2 m. You may assume that $0.1 \leq r \leq 4$ m. Use the random number table provided to you. The “initial temperature T” (*used in* *the algorithm that does not correspond to the physical temperature in this problem*) may be taken to be the average of four objective function values for appropriately chosen values of the radius. (9)

(8) A supersonic military aircraft engine typically employs an after burner after the combustor where additional fuel is injected in order to increase the thrust. The total energy, E, at the exit of the afterburner is to be maximized for maximum work output. One such aircraft is flying at 600 m/s and intakes air at 10.5 kg/s (C_p for air is 1005 J/kgK). The temperature at the inlet of the engine is -25°C and the calorific value of the fuel is 44000 kJ/kg. The fuel flow rate to the combustor is x_1 kg/s and that to the after burner is x_2 kg/s. Due to spraying limitations, a constraint in the form of $8 x_1 + 6 x_2 \leq 3$ arises. The combustor can withstand more heat than the afterburner and so $x_1 - x_2 \geq 0.1$. Furthermore, due to limitations in fuel storage and distribution $2 x_1 + 5 x_2 \leq 1.2$.

(a) Set up the optimization problem for maximizing energy at the exit (Note: Due to high velocities, kinetic energy term may not be negligible!) of the afterburner as a Linear Programming (LP) problem. (4)
(b) Solve the LP using the graphical method. (6)
(c) Solve the LP using the method of slack variables and compare the solutions obtained in (b) and (c). (4)

(9) Consider a container truck that is climbing a "ghat" (hill) road. There are three sections on the ghat road denoted by A-B, B-C and C-D. The fuel consumed in each section varies with the time taken to cover the particular section and is tabulated below. Please note that section BC is the toughest part of the ghat road that "drinks" so much of fuel. The hill climb needs to be completed within a total time of 34 s. Using dynamic programming, determine the optimal time to be spent by the truck in the three sections so as to minimize the total fuel consumption. What is the computational gain achieved by using the dynamic programming for this problem?

Section	Time, t, s	Fuel consumption, g
A-B	10	60
	11	51
	12	43.5
	13	37.5
B-C	10	91.5
	11	78
	12	67.5
	13	57
C-D	10	73.5
	11	61.5
	12	52.5
	13	45

(8)