Vibrations of Structures

Module V: Vibrations of Plates

Exercises
1. A square plate of side $a$ is simply-supported at the four edges, and carries a particle of mass $m$ at the center. Determine the eigenfrequencies and eigenfunctions of the plate.

2. A circular plate of radius $a$ is simply supported at the boundary. Determine the dynamic reaction forces at the boundary for different modes of vibration of the plate.

3. An elliptic plate of semi-major axis $a$ and semi-minor axis $b$ is simply supported at the boundary. Determine the approximate eigenfrequencies and modes of vibrations. Plot the variation of the first six eigenfrequencies with the ratio $a/b$ in the range $(1, 2)$.

![Figure 1: Exercise 4](image1.png)

4. A circular plate of radius $a$ is simply-supported on a circle of radius $b$, as shown in Fig. 1. Determine the optimum ratio $b/a$ for which the plate is most firmly supported in the mode $(0,1)$ (i.e., the corresponding frequency is maximized).

5. A circular plate of radius $a$ is clamped at the boundary $r = a$. A particle of mass $m$ is dropped from a height $h$ exactly on the center of the plate. The particle sticks to the plate. Determine the motion of the plate and the force between the particle and the plate.

![Figure 2: Exercise 6](image2.png)

6. An annular plate of inner radius $a$ and outer radius $b$ is clamped at the boundary $r = b$, and clamped to a massless collar (at $r = a$) sliding without friction on a guide, as shown in Fig. 2. Determine the eigenfrequencies and eigenfunctions of the system. If the collar is excited by a harmonic force $Q(t) = A \cos \Omega t$, determine the response of the plate.
7. A circular plate of radius $a$ is clamped at the boundary. A constant point force is traveling on a circular path around the center of the plate at a radius $r_0$, i.e., $q(r, \phi, t) = Q_0 \delta(r-r_0) \delta(\phi-\Omega t)$, where $Q_0$ is the constant magnitude, and $\Omega$ is the angular speed. Determine the response of the plate. At what values of $\Omega$ will the plate resonate?

8. A square plate of side $a$ is simply supported at the edges on a rigid frame. The frame is given harmonic angular oscillations of circular frequency $\Omega$ about a center line parallel to an edge. Determine the response of the plate.