Vibrations of Structures

Module V: Vibrations of Plates

Exercises
1. A square plate of side $a$ is simply-supported at the four edges, and carries a particle of mass $m$ at the center. Determine the eigenfrequencies and eigenfunctions of the plate.

2. A circular plate of radius $a$ is simply supported at the boundary. Determine the dynamic reaction forces at the boundary for different modes of vibration of the plate.

3. An elliptic plate of semi-major axis $a$ and semi-minor axis $b$ is simply supported at the boundary. Determine the approximate eigenfrequencies and modes of vibrations. Plot the variation of the first six eigenfrequencies with the ratio $a/b$ in the range $(1, 2)$.

4. A circular plate of radius $a$ is simply-supported on a circle of radius $b$, as shown in Fig. 1. Determine the optimum ratio $b/a$ for which the plate is most firmly supported in the mode $(0, 1)$ (i.e., the corresponding frequency is maximized).

5. A circular plate of radius $a$ is clamped at the boundary $r = a$. A particle of mass $m$ is dropped from a height $h$ exactly on the center of the plate. The particle sticks to the plate. Determine the motion of the plate and the force between the particle and the plate.

6. An annular plate of inner radius $a$ and outer radius $b$ is clamped at the boundary $r = b$, and clamped to a massless collar (at $r = a$) sliding without friction on a guide, as shown in Fig. 2. Determine the eigenfrequencies and eigenfunctions of the system. If the collar is excited by a harmonic force $Q(t) = A \cos \Omega t$, determine the response of the plate.
7. A circular plate of radius $a$ is clamped at the boundary. A constant point force is traveling on a circular path around the center of the plate at a radius $r_0$, i.e., $q(r, \phi, t) = Q_0 \delta(r-r_0) \delta(\phi-\Omega t)$, where $Q_0$ is the constant magnitude, and $\Omega$ is the angular speed. Determine the response of the plate. At what values of $\Omega$ will the plate resonate?

8. A square plate of side $a$ is simply supported at the edges on a rigid frame. The frame is given harmonic angular oscillations of circular frequency $\Omega$ about a center line parallel to an edge. Determine the response of the plate.