Vibrations of Structures

Module IV: Vibrations of Membranes

Exercises
1. Determine the eigenfrequencies and modes of vibration of a right isosceles triangular membrane of hypotenuse \( \sqrt{2}a \) that is fixed at all the boundaries.

2. Determine the eigenfrequencies and modes of vibration of an annular membrane that is fixed at the boundaries \( r = a \) and \( r = a/2 \).

![Figure 1: Exercise 3](image1.png)

3. A rectangular membrane of width \( a \) and length \( 2a \) is made of two materials of mass densities \( \mu_1 \) and \( \mu_2 \) which are joined together, as shown in Fig. 1. Derive the characteristic equation and determine the eigenfrequencies and eigenfunctions.

4. A circular membrane of radius \( a \) with fixed boundary has a small particle of mass \( m \) attached at the center. Estimate the eigenfrequencies of the membrane.

![Figure 2: Exercise 5](image2.png)

5. An annular membrane of density \( \mu \) and uniform tension per unit length \( T \) is fixed at the outer radius \( b \), and connected to a thin ring of mass \( m \) and radius \( a \), as shown in Fig. 2. Determine the approximate eigenfrequencies and modes of vibrations of the system.

6. A composite circular membrane over a hemispherical enclosure (a kettledrum) consists of a central circular membrane of radius \( a \) and mass density \( \mu_1 \), and an annular membrane of mass density \( \mu_2 \) between the radii \( a \) and \( b \). Assuming a uniform tension per unit length \( T \), determine the approximate eigenfrequencies and modes of vibrations of the drum.