Vibrations of Structures

Module II: Wave Propagation and Scattering

Exercises
1. A uniform homogeneous bar is under tension due to a string, as shown in Fig. 1. If the string suddenly snaps, determine the transient motion and stress waves set-up in the bar.

2. An infinite string at rest is excited by a force \( q(x,t) = F(t)\delta(x) \). Determine the subsequent motion of the string when (a) \( F(t) = F_0\delta(t) \) (impulse), and (b) \( F(t) = F_0\mathcal{H}(t) \) (Heaviside step). (Use Fourier transform for space and Laplace transform for time.)

3. A semi-infinite string is connected at \( x = 0 \) to a spring-mass system as shown in Fig. 2. At \( t = 0 \), a waveform given by

\[
 f(\xi) = \begin{cases} 
 A \left( 1 - \cos \frac{2\pi \xi}{l} \right), & \xi \in [0,l] \\
 0, & \xi \geq l
\end{cases}
\]

is incident at \( x = 0 \) from the right. Analyze the wave refection process when (a) \( m = 0 \) and \( k \neq 0 \), (b) \( m \neq 0 \) and \( k = 0 \), and (c) \( m \neq 0 \) and \( k \neq 0 \). In case (c), what happens if the incident wave is resonant, or non-resonant?

4. Two semi-infinite bars of different materials and diameters are joined, as shown in Fig. 3. A positive traveling longitudinal wave \( f_I(x - c_1t) \) in the left bar is incident on the junction at \( x = 0 \). Determine the reflected and transmitted waves \( f_R(x + c_1t) \) and \( f_T(x - c_2t) \), respectively.
5. Analyze the wave scattering process in Exercise 4 if a thin damping material (modeled as a discrete dashpot with viscous damping coefficient $d$) is introduced at the junction between the two bars in Fig. 3. Define $\eta = |C_R|^2 + |C_T|^2$ as the fraction of the average incident power after the scattering process. For what value of $d$ will $\eta$ be minimized.

6. An infinite string is provided a support with internal damping, as shown in the Fig. 4. A positive traveling harmonic wave is incident from the left. Determine $d$ for maximizing the absorption of the incident power by the support. (Minimize the function $\eta$ defined in Exercise 5.)

7. A uniform homogeneous bar of length $l$, density $\rho$, and section-modulus $EA$ is subjected to an axial force of the form $F(t) = F_0[H(t) - H(t - \tau)]$, where $\tau$ is a constant, as shown in Fig. 5. Determine the motion of the bar in terms of the traveling elastic waves set-up inside it when $0 < \tau < 2l/c$ and $2l/c < \tau < 4l/c$. 