PID Controller Design – Part B

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This Lecture Contains

- How to tune a PID Controller?
- Zeigler-Nichol’s Rules
- Simulation
- A Special Rule for First Order System
How do we tune a PID Controller?

• In the last lecture, we have shown how by choosing the three different tuning constants of the PID compensator you can control the nature of the response. However, the question remains how can we obtain the actual values of these constants. Is it only through trial and error or are there any rules to obtain them?

• Sometimes in Industry people try to tune compensators intuitively. For example, we do know that the derivative constant helps to remove sluggish response of a system or the integral constant helps to remove offset errors. However, quite often it is found that these constants are interrelated. For example, increasing integral constant may help to improve the steady state response but it may increase the system overshoot.

• A popular rule for tuning which is being used since last century is known as Zeigler-Nichols rule. This was developed way back in 1942 and is still popular today.
Zeigler-Nichols Rule

- There are two sets of rules. These rules are based on transient response of the system. The system dynamics may or may not be known to us. Let us now consider the control output $U(s)$ to be defined in terms of the tuning constants as follows:

$$U(s) = K_p \left[ 1 + \frac{1}{T_i s} + T_d s \right] E(s)$$

- The first rule is for systems for which the exact dynamics is unknown.
- Using only a proportional controller first increase the gain so much that the response of the system starts to show oscillatory behavior. The corresponding gain could be termed as $K_0$ and the corresponding time period as $T_0$.
- Now the constants are determined by using the following table-
The Tuning Table for PID Controller

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>$K_p$</th>
<th>$T_i$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$0.5K_0$</td>
<td>$\infty$</td>
<td>0</td>
</tr>
<tr>
<td>PI</td>
<td>$0.45K_0$</td>
<td>$1/1.2T_0$</td>
<td>0</td>
</tr>
<tr>
<td>PID</td>
<td>$0.6K_0$</td>
<td>$0.5T_0$</td>
<td>$0.125T_0$</td>
</tr>
</tbody>
</table>

The transfer function corresponding to the PID controller may be written as

$$T(s) = 0.07K_0T_0 \left(\frac{s + 4/T_0}{s}\right)^2$$
Zeigler-Nichol’s tuning for Known Systems

• Now consider a dynamic system for which the system parameters are known and hence mathematical modeling is available.

• Consider the system where $T_i = \infty$ and $T_d = 0$. Now for such a system, find out by using Routh’s stability analysis the critical gain $K_0$ for which the roots cross the $j\omega$.

• Find out the frequency and the time period of the oscillation. Then, use the table mentioned earlier to determine the constants.

• These should be used as the starting point for tuning. Obtain the step response of the system and check whether the overshoot is less than 25%. If not, you should fine tune the system by moving the double zeros introduced by the PID controller.
Simulation Example

Let us consider a 2\textsuperscript{nd}-order system having two real poles at -1 and -3. The step response of the over-damped system is shown below:
Next step to find the Critical Gain:

- Let us look at the root-locus of the system. At unit gain, the system will be just critically damped beyond which it will start to oscillate at 2 rad/s frequency. Hence, $K_0 = 1$ and $T_0 = 0.5$. 

![Root Locus Diagram](image)

System: tf1
Gain: 1
Pole: -2
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 2
PID Controller following Z-N Rule:

• Following the Z-N Table, the controller transfer function may be written as:

\[ T(s) = 0.035 \frac{(s + 8)^2}{s} \]

The resultant system is found to have increased robustness and at gain 233, the overshoot is about 25% which is quite acceptable.
A Special Zeigler-Nichol’s Rule for First Order System Behavior

• Let us consider a first order system which may be characterized as follows:

• Here, L is known as the time delay and T the time constant. The control gain could be evaluated in terms of these parameters as shown in the following table.
First Order System – A Special Table

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>$K_p$</th>
<th>$T_i$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$T/L$</td>
<td>$\infty$</td>
<td>0</td>
</tr>
<tr>
<td>PI</td>
<td>$0.9T/L$</td>
<td>$L/0.3$</td>
<td>0</td>
</tr>
<tr>
<td>PID</td>
<td>$1.2T/L$</td>
<td>$2L$</td>
<td>$0.5L$</td>
</tr>
</tbody>
</table>
Special References for this lecture

- Feedback Control of Dynamic Systems, Frankline, Powell and Emami, Pearson
- Control Systems Engineering – Norman S Nise, John Wiley & Sons
- Design of Feedback Control Systems – Stefani, Shahian, Savant, Hostetter
  Oxford